The KITT's Initial Contribution to the Educational Activities at Budapest Tech

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Abstract: The Transportation Informatics and Telematics Knowledge Centre (after the abbreviation of its name in Hungarian: 'KITT') of Budapest Tech initiated its operation in the Autumn of 2006 thank to the financial support provided by the 'National Office for Research and Technology' (NKTH) and the 'Agency for Research Fund Management and Research Exploitation' (KPI) using the resources of the 'Research and Technology Innovation Fund' within the project No. RET-10/2006. One of the duties of KITT is to give contribution to the educational activities of the host institution of the research consortium, which role is played in this case by Budapest Tech. Regarding education, the main aim is transferring the freshest research results achieved by KITT into the education without considerable delay. Following the first few months of KITT's activities, Bánki Donát Faculty of Mechanical and Safety Engineering initiated its BSc courses in which, at first time, within the sub-branch of 'Robot Systems' of the branch of 'Mechatronics', the course 'Control of Robots' was launched in the Autumn semester of the academic year 2007/2008. Due to a lucky constellation in the KITT's Project No. 2.3 entitled 'Automatic Analysis of Vehicle Behavior' partly is involved the automatic observation, analysis and control of strongly coupled nonlinear systems of which normally very limited, imprecise, and incomplete 'a priori' information is available for the controller, as e.g. in the case of *controlling platoons. This situation is quite typical in control of robots, too, for which the study of standard robust approaches as the 'Robust Sliding Mode / Variable Structure (VS/SM)' and adaptive techniques as 'Adaptive Inverse Dynamics (AID)' or the adaptive algorithm elaborated by Slotine and Li can be designed on the basis of Lyapunov's 2nd Method. In the present paper the part of the curriculum is discussed in which the operation of the VS/SM, the AID, and Slotine's and Li's adaptive approaches are compared with that of the novel method elaborated at Budapest Tech (Fixed Point Transformations Based Adaptive Control) by the use of a very simple paradigm.*

1 Challenges in Teaching Control of Robots

During their education the present students of the BSc course up to this point have obtained various knowledge in a not very 'integrated' manner. A part of this knowledge is mainly of lexical nature strictly related to practical applications, the other part is rather devoted to develop a kind of 'inductive thinking' also supporting the solution of practical problems, while the most 'theoretical' segment is related to Mathematics that scarcely is connected to real applications. The students have some superficial idea of linear operators, linear algebra, matrix product and determinant, the main rules of derivation and integration without having any practice in applying these rules. Similar conditions hold regarding their ideas concerning the relationship between the various models of the reality and the 'physical reality itself'. For instance, some of them are not aware at all of the fact that several program or software blocks used for quantitative calculations hide or contain certain models of the reality. These students are apt to believe that the numerical values used or provided by the software blocks '*immediately belong to the reality*' and know nothing of the fact that *these quantities are connected to the reality only through certain models*. The idea that '*the same reality*' can be described by various models of various levels of abstraction at a first glance seems to be unbelievable for them.

After summarizing the experiences obtained by teaching a whole semester devoted to '*Kinematics and Dynamics of Robots*', and the first half of the semester '*Control of Robots*' it cropped up that these subject areas are the first ones in their studies that require a) appropriate skill for abstraction, and b) certain skill for the application of the fundamental mathematical rules in the practice. In spite of the fact that this observation is disappointing and does not shed nice light on the quality of education in the secondary school level institutions, as a teacher, one has to cope with this difficulty and find a way out of the main problem. In the sequel the main point of this idea is briefly reported.

2 Geometric Way of Thinking as Probably the Best Possible General Solution

Considering the historical antecedents of geometric way of thinking in natural sciences makes one persuaded that until the 1st half of the 20th century the development of Mathematics aimed at serving the needs of natural and technical sciences. In the history of the '*quantitative sciences*' geometric way of thinking always played a pioneering rule. The principles of geometry first were reduced to a small set of axioms by *Euclid of Alexandria*, a Greek mathematician who worked during the reign of Ptolemy I (323-283 BC) in Egypt. His method of proving mathematical theorems by logical reasoning from accepted first principles remained the backbone of mathematics even in our days, and is responsible for that field's characteristic rigor.

Following the pioneering work clarifying the phenomenology of *Classical Mechanics* by Galilei and Newton, in his fundamental work entitled '*Mécanique Analytique'* [1] Joseph-Louis Lagrange (1736-1813) solved various optimization problems under constraints, introduced the concept of '*Reduced Gradient*' and that of what we refer to nowadays as '*Lagrange Multipliers*'. It has to be noted that at that time the concept of '*linear vector spaces*' was not clarified at all. The first mathematical means of describing quantities with direction, i.e. the '*quaternions*' introduced by *Sir William Rowan Hamilton* (1805-1865) appeared not very long time after Lagrange's death [2]. In the 19th century quaternions were generally used for such purposes. For instance, in the first edition of Maxwell's famous '*Treatise on Electricity and Magnetism*' quaternions were used for describing the 'directed' magnetic and electric fields [3].

The first known appearance of what are now called '*linear algebra*' and the notion of *vector spaces* is related to *Hermann Günther Grassmann* (1809-1877), who started to work on the concept from 1832. In 1844, Grassmann published his masterpiece [4] that commonly is referred to as the 'Theory of Extension' or '*Theory of Extensive Magnitudes*'. This work was mainly inspired by Lagrange's '*Mécanique Analytique'*. Grassmann showed that once geometry is put into the *algebraic form* he advocated, then the number three has no privileged role as the number of spatial dimensions: the number of possible dimensions is in fact unbounded.

The close relationship between geometry and algebra was realized and strongly utilized by *William Kingdon Clifford* (1845-1879) who introduced various *associative algebras*, the so called '*Clifford Algebras*'. As special cases Clifford Algebras contain the algebra of the real, the complex, the dual numbers, the quaternion algebra, and the algebra of octonions (biquaternions) [5]. His *Geometric Algebra* is widely used in technical sciences as e.g. in computer graphics, robotics, etc. Clifford was the first to suggest that gravitation might be a manifestation of an underlying *Riemannian Geometry*.

Equipped with the concepts of linear vector spaces *Marius Sophus Lie* (1842-1899) in his PhD dissertation studied the properties of geometric symmetry transformations [6]. One of his greatest achievements was the discovery that continuous transformation groups (now called after him '*Lie Groups*') could be better understood by studying the properties of the tangent space of the group elements, that form linear vector spaces (the vector space of the so-called infinitesimal generators), and with the commutator as multiplication also form algebras, the so called '*Lie Algebras*'.

In the very fertile period of Mathematics, in the 19th century *Georg Friedrich Bernhard Riemann* (1826-1866) elaborated the *geometry of curved spaces* in a special form that made it possible to study physical quantities as tensors even if the geometry of the space differs from the *Euclidean Geometry*. This concept was very fruitfully used in the *General Theory of Relativity*.

Figure 1
The advantages of geometric way of thinking: it makes it possible to apply lucid and simple problem formulation and argumentation with which we have became familiar already in our childhood on quite 'abstract' fields generating the feeling of 'cozy familiarity'

David Hilbert (1862-1943) extended the concept of the Euclidean Geometry to linear, normed, complete metric spaces in which the norm originates from a scalar product. His invention had extreme advantages in Physics and technical sciences since it makes it possible to apply a way of geometric thinking with which we became familiar from our childhood in the daily experienced *Euclidean Geo*metry of the reality around us (Fig. 1). *Stefan Banach* (1892-1945) introduced the more general concept, the concept of *Banach Spaces*, that are linear, normed, complete metric spaces in which the norm not necessarily originates from a scalar product.

The great practical advantage of Banach's invention is that by adding various norms to the same mathematical set various complete, linear, normed metric spaces can be obtained that offer a wide basis for elaborating diverse practical variants and solutions pertaining to the essentially same basic idea. *Vladimir Igorevich Arnold* (1937-) studied the *Symplectic Geometry* and *Symplectic Topology* that are extremely useful means of studying the behavior of various Mechanical and other physical systems.

Our aim with providing this brief historical survey was to show that geometric way of thinking is a very useful and fruitful mode of problem-tackling in various fields as e.g. in technical sciences. In the sequel its advantages will be shown in the field of nonlinear control.

3 Geometric Way of Thinking in Classical Mechanics

The students meet a rigorous, abstract, and fully deductive treatment in connection with modeling the kinematics and dynamics of robots. Kinematic modeling gives good opportunity for introducing and using *Group Theory* and *Lie Groups* in the proper ‘parameterization’ of the possible transformations. The main principles of *Classical Mechanics (CM)* can be formulated by using various ‘minimum principles’. The most successful approach is the *Hamilton Principle* that is related to the phenomenology of CM established by Galilei and Newton via the *Lagrangian*: the Lagrange Function must be determined by using the Cartesian coordinates of the velocities with respect to an *inertial system of reference*. This leads to a clear phenomenological interpretation of the ‘*generalized forces*’ in the so called *Euler-Lagrange Equations of Motion* that systematically can be constructed for an arbitrary system. Though these equations are quite satisfactory for use in practical control issues, they do not convey immediate geometric interpretation. However, it is not worthless to show the students that by the use of the *Legendre Transformation* the so called *Canonical Equations of Motion* can be obtained. Besides having direct geometric interpretation this formalism reveals very profound statements for isolated systems as ‘trivialities’ as a) the conservation of energy, b) the relationship between internal symmetries and conservation rules, c) the fact that the equations of motion canonically map the state space onto itself, and d) the ‘flow’ generated by the equations of motion is similar to the flow of incompressible fluids. (Further properties of the *Symplectic Transformations* can be utilized in stability analysis of parametric resonance phenomena later.)

4 Geometric Interpretation of Lyapunov's 2^{nd} Method

Lyapunov's 2^{nd} Method is a widely used technique in the analysis of the stability of the motion of the *non-autonomous dynamic systems* of equation of motion as $d\mathbf{x}/dt=\mathbf{f}(\mathbf{x},t)$. The typical stability proofs provided by Lyapunov's original method published in 1892 [7] (and later on e.g. [8]) have the great advantage that they do not require to analytically solve the equations of motion. Instead of that the *uniformly continuous nature* and non-positive time-derivative of a positive definite Lyapunov-function V constructed of the tracking errors and the modeling errors of the system's parameters are assumed in the $t\in[0,\infty)$ domain from which the convergence $dV/dt\to 0$ can be concluded according to Barbalat's lemma [9]. This lemma states that if the integral of a uniformly continuous function (in this case the integral of dV/dt i.e. V) in $[0,\infty)$ exists and is finite then this function has to converge to zero [9]. As in the case of any self-consistent approach based method this assumption has to be verified later. The uniform continuity of dV/dt used to be guaranteed by showing that d^2V/dt^2 is bounded. Due to the positive definite nature of V from that it normally follows that the tracking errors have to remain bounded, or in certain special cases, has to converge to 0.

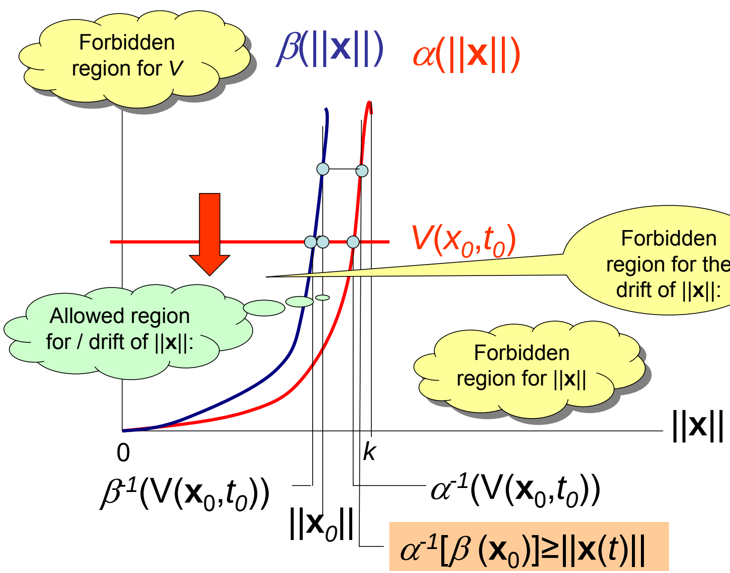


Figure 2

Geometric interpretation of Lyapunov's 2^{nd} Method by using the functions of '*class κ*': if $\alpha(||\mathbf{x}||)$ serves as a lower bound for a stagnating or decreasing V, the state is evidently stable; if $\alpha(||\mathbf{x}||)$ and $\beta(||\mathbf{x}||)$ serve as a lower and upper bounds for a stagnating or decreasing V, the state is evidently uniformly stable; if $\alpha(||\mathbf{x}||)$ and $\beta(||\mathbf{x}||)$ serve as a lower and upper bounds for a decreasing V that achieves zero at $t\to\infty$ the state is evidently asymptotically stable

'Less abstract' and trivial proof of Lyapunov's 2nd Method can be obtained by using the functions of '*class κ*' by considering the horn- or funnel-shaped region in Fig. 2 with stagnating or decreasing Lyapunov function *V*. In spite of its lucid and simple geometric interpretation the application of Lyapunov's 2nd method in the practice is rather an 'art' than some mechanically applicable tool. It requires not only great practice but also needs good intuition. Furthermore, the computational need of realizing the method is not always negligible. That makes its use difficult in education is the fact that the Lyapunov function need not to be constructed within the computer program used for the control: only its consequences have to be utilized that may make this topic a little bit 'mysterious' for the students. However, its use cannot be evaded in teaching the *AID control* and *Slotine's and Li's adaptive approaches*. Alternatively, in the *VS/SM control* and in the novel geometric approach developed at Budapest Tech no Lyapunov function has to be used.

5 The Novel Geometric Approach

Regarding the technical realization of the so outlined program was the introduction of an '*Excitation - Response Scheme*'. According to that scheme each control task could be formulated by using the concepts of the appropriate '*excitation*' **Q** of the controlled system to which it is expected to respond by some prescribed or '*desired response*' $\mathbf{r}^d$. The physical meaning of the appropriate excitation and response depend on the phenomenology of the system under consideration. In the case of Classical Mechanical Systems the excitation physically can be force and/or torque, while the response can be linear or angular acceleration, etc. The appropriate excitation can be computed by the use of some '*available approximate inverse dynamic model*' as $\mathbf{Q}=\boldsymbol{\varphi}(\mathbf{r}^d)$. Since normally this inverse model is neither complete nor exact, the actual response determined by the system's dynamics, $\boldsymbol{\psi}$, results in a '*realized response*' $\mathbf{r}^r$ that differs from the desired one: $\mathbf{r}^r=\boldsymbol{\psi}(\boldsymbol{\varphi}(\mathbf{r}^d))\neq\mathbf{r}^d$. It is worth noting that the functions $\boldsymbol{\varphi}()$ and $\boldsymbol{\psi}()$ may contain various hidden parameters that partly correspond to the dynamic model of the system, and partly pertain to unknown external dynamic forces acting on it. Due to phenomenological reasons the controller can manipulate or '*deform*' the input value from rd to some $\mathbf{r}^{*d}$ so that $\mathbf{r}^d=\boldsymbol{\psi}(\boldsymbol{\varphi}(\mathbf{r}^{*d}))$. Other possibility is the manipulation of the output of the rough model. The above structure evidently indicated that using the pairs of the '*desired*' response known and set by the controller and comparing it to the observed/observable 'realized' response mathematically can be formulated as seeking the solution of a Fixed Point Problem. From this point on the main direction of the research was seeking various deformations or fixed point transformations that were able to generate appropriate sequences of responses that can converge to the fixed point. In this approach in each control cycle one iterative step can be done with the actually available updated '*desired response*', and in the next cycle the deformation applied can be updated on the basis of the '*observed response*'. Supposing that the dynamics of the adaptive iteration is considerably faster than the variation of the control task determined e.g. by the nominal trajectory to be tracked such solution may result in practically acceptable tracking. From the set of the various solutions we elaborated let us show one possible transformation for SISO systems (Fig. 3).

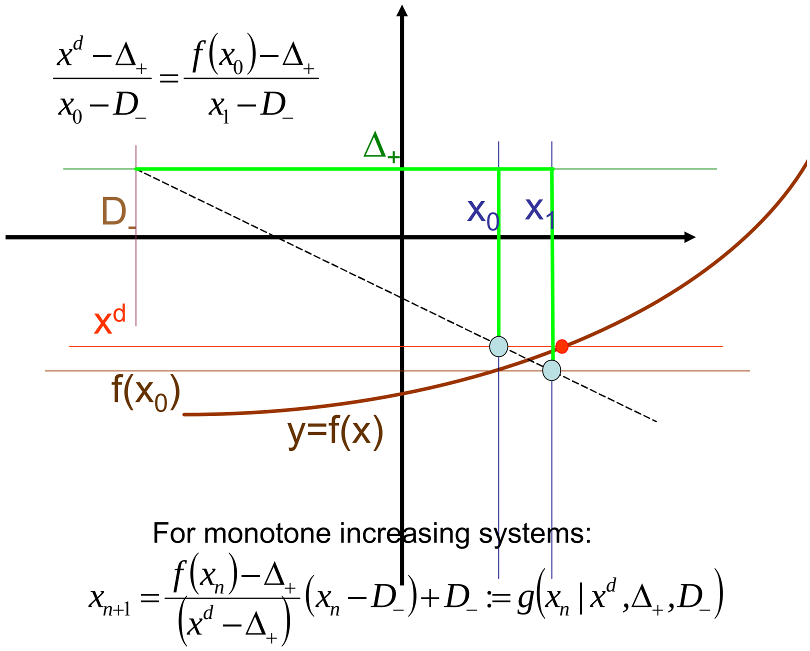


Figure 3

Geometric interpretation of a special fixed point transformation for monotone increasing SISO systems

What 'intuitively' is indicated by this figure, i.e. that for flat function f the sequence obtained by the iteration specified can be more generally proved for MIMO case using the concept of contractive maps in *Banach Spaces*.

$$
\begin{aligned}
&g(x\,|\,x^d,D_-,\Delta_\pm):=\frac{f(x)-\Delta_+}{x^d-\Delta_+}(x-D_-)+D_- \\
&g'(x)=f'(x)\frac{x-D_-}{x^d-\Delta_+}+\frac{f(x)-\Delta_+}{x^d-\Delta_+} \\
&g'(x_*)=f'(x_*)\frac{x_*-D_-}{x^d-\Delta_+}+1
\end{aligned}
\tag{1}
$$

According to (1) if $f(x_*)=x^d$ then $g(x_*)=x_*$, furthermore by simply setting the parameters D_- and Δ_+ g evidently can be made contractive (i.e. achieving $|g'|<1$) in the vicinity of x_* that means the possibility of constructing a simple iterative adaptive control. In the sequel the teaching method will be described and certain comparative results will be provided.

6 The Deductive Teaching Method

The essence of the deductive way of thinking is that general schemes can be considered, explained/proved for use in wide problem classes by the utilization of certain quite general properties. Then a particular, concrete problem can be considered in the case of which the particular representatives of the general schemes has to be recognized. For this purpose at first the simplest example, a SISO system, a mass point [m kg] plus elastic spring [k N/m] system can be considered under the effect of gravitation [g m/s^2]. For instance, in the adaptive version of Slotine's and Li's method the generalized forces that are exerted to the actual system are calculated by an imprecise model of exact analytical form in which $\mathbf{e}=\mathbf{q}-\mathbf{q}^N$ is the tracking error, $\mathbf{\Lambda}$, $\mathbf{K}_D$ have to be positive definite symmetric matrices, and the 'cap' (^) refers to the model values. The '*general*' and the '*special*' forms of the forces to be exerted are given in (2) and (3), respectively.

$$\mathbf{Q}=\hat{\mathbf{H}}(\mathbf{q})\left(\underbrace{\ddot{\mathbf{q}}^N-\Lambda\dot{\mathbf{e}}}_{\dot{\mathbf{v}}}\right)+\hat{\mathbf{C}}\left(\underbrace{\dot{\mathbf{q}}^N-\Lambda\mathbf{e}}_{\mathbf{v}}\right)+\hat{\mathbf{g}}-\mathbf{K}_D\left(\underbrace{\dot{\mathbf{e}}+\Lambda\mathbf{e}}_{:=\mathbf{r}}\right) \quad (2)$$

$$Q=\hat{m}\left[\ddot{x}^N-\Lambda\left(\dot{x}-\dot{x}^N\right)\right]+\hat{k}x+\hat{m}\hat{g}-\hat{k}x_0-K_D\left[\dot{x}-\dot{x}^N+\Lambda\left(x-x^N\right)\right] \quad (3)$$

For calculating the parameter updating rule the modeling errors have to be considered in its general, and its special form as

$$\tilde{\mathbf{H}}(\mathbf{q})\dot{\mathbf{v}}+\tilde{\mathbf{C}}\mathbf{v}+\tilde{\mathbf{g}}=\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\mathbf{v},\dot{\mathbf{v}})\tilde{\mathbf{p}} \quad (4)$$

$$\tilde{m}\dot{v}+\tilde{k}x+\overbrace{(mg-kx_0)}^{\sim}=\begin{bmatrix}\dot{v} & x & 1\end{bmatrix}\begin{bmatrix}\tilde{m}\\ \tilde{k}\\ \overbrace{(mg-kx_0)}^{\sim}\end{bmatrix} \quad (5)$$

that identifies the $\mathbf{Y}$ and $\mathbf{p}$ arrays in this special case. Similar procedure can be applied in the case of the *AID* method, the *VS/SM* control, and in the case of the application of fixed point transformations. Though these methods have quite different parameters so possible different setting of which cannot 'strictly be compared' to each other, typical features of the operation can be compared. The computations were made by INRIA's SCILAB 4.1 software though for the purposes of education the MS Office is an excellent tool, too [10], [11].

As it can be seen in Fig. 4 describing the trajectory tracking achieved by the *AID* method is relatively sensitive to the effect of external perturbations, while other ones seem to be more robust. More illustrative details are revealed by the graphs describing directly the tracking errors and the phase trajectories (Fig. 5).

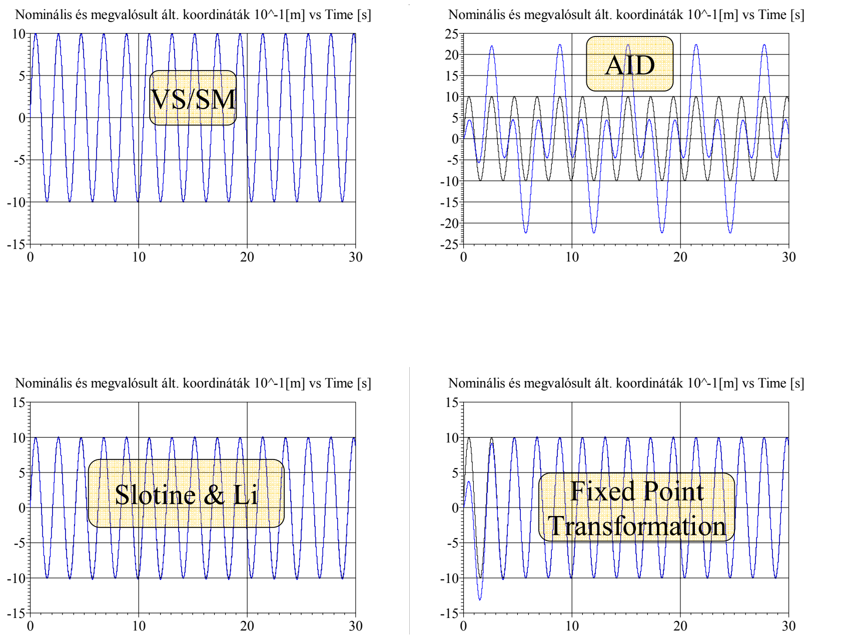


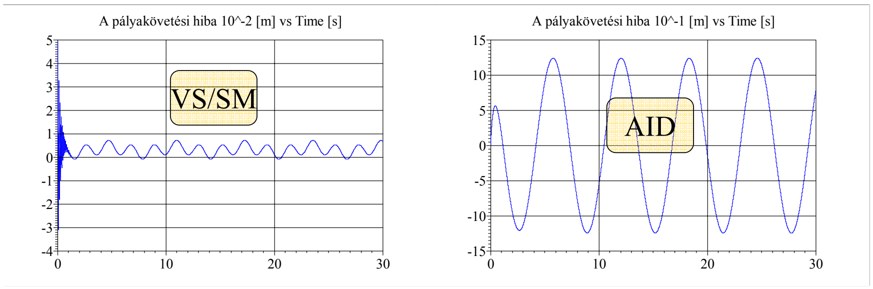
Figure 4

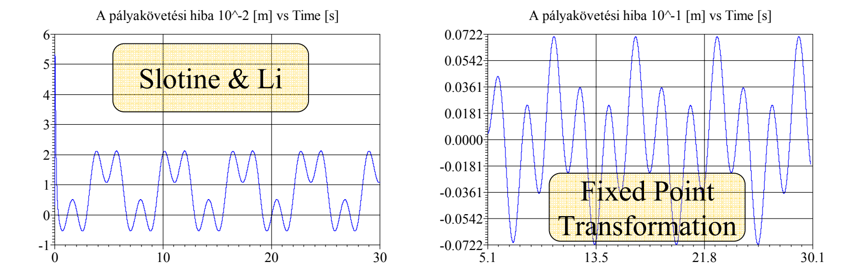
Comparison of trajectory tracking for significant slow external perturbation

Conclusions

In this paper the KITT's contribution to the educational activities of Budapest Tech was briefly summarized on the basis of the experiences based on the first half of the semester regarding the module '*Control of Robots*'. It was found that the problems that have to be solved by KITT's co-workers are strongly nonlinear problems with considerable coupling effects. Efficient tackling of such problems require some skill for working at various levels of abstraction, as well as sound understanding of the fundamental control principles. It was found that our students are in general lack of skills for abstraction, and they also need more sound fundamental, application-oriented mathematical knowledge, and some programming experiences, since for verifying the ideas in the realm of nonlinear systems instead of using closed analytical formulae numerical computations are the only available tools.

Regarding abstraction, on the basis of a brief historical overview the geometric approach was proposed. It was shown that the more traditional non-linear control tricks as well as the novel method developed at Budapest Tech can easily, lucidly, and well be tackled as geometric problems.





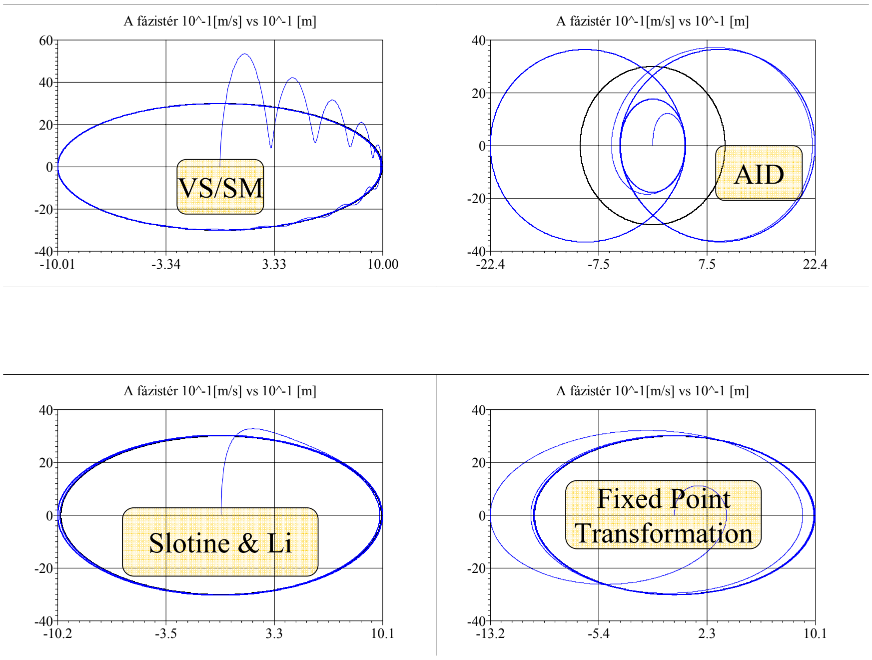


Figure 5

Tracking errors and phase trajectories for significant slow external perturbation

The other important point is that the students have to learn how to identify particular representations of more general formulae in the case of a given problem to be solved. Such a skill can improved simultaneously with programming skills by using 'sample programs' solving a given problem by a well defined method. This program can be copied and modified for solving the same problem with another control method.

Following the experiences obtained in the first half of the actual semester it can be stated that this educational task is not hopeless. We also think that revealing the possibility of working with such a way of thinking is our moral duty for improving professional development of our students. In a more healthy system they should have been provided with similar ideas much earlier.

Acknowledgement

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