# Stochastic Approach to the Fuzzyfication of the Input Signal Quantities

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*Abstract: In classical measurement in the whole measurement range absolute error is, more or less, the same, but in the case of control we know that all measurement subintervals are not equally significant. Stochastic adding A/D conversion (SAADK) represents a method of measurement which naturally can replace process of fuzzyfication of input quantities, depending on the choice of frequency of reading, therefore the membership on the universe is time-depending and with random uncertainties. Those memeberships of continuous set are ordered to the membership function values as a secondary fuzzy set, constructed a type-2 fuzzy environment The distance based operators on type-2 fuzzy sets, which are defined using min and max operators will give an opportunity to simply hardware realization of the stochastic adding A/D conversion for signals.*

*Keywords: stochastic A/D conversion, fuzzy distance based operators, type-2 fuzzy sets*

## 1 Introduction

The developed measure method SAADK stochastic adding A/D conversion has several advantages compared to standard digital instrumentation. Its main advantages are extremely simple hardware and, consequently, simple implementation of parallel measurements, and on the other hand possibility to trade speed for accuracy [1]. This instrument can be either fast and less accurate, or slow and very accurate, depending on the choice of frequency of reading its output, therefore the membership of the signal on the universe *X* is time-depending and with random uncertainties. In the paper the SAADK method is shortly represented.

The distance based operators [2] applied on type-2 fuzzy sets (T2 FS) are first of all preliminary works to construct further hardware realizations of inference mechanism based on this family of fuzzy operators and T2 fuzzy logic using SAADK. Furthermore those processes are related generally to the signal processing. The SAADK instrument can be either fast and less accurate, or slow and very accurate, depending on the choice of frequency of reading its output, therefore the membership on the universe *X* is time-depending and with random uncertainties, it means suitable to represent it as T2 FS. The hardware realization of the problem is based only on the min and max operators, therefore distance based operators, which are defined using min and max operators will be introduced in the process of realization. It is an opportunity to use simply hardware realization of the operators and approximate reasoning process in the control problems based on the SAADK method.

# 2 Type-2 Fuzzy Sets and the Distance-based Fuzzy Operator Group

L. A. Zadeh introduced type-2 and higher-types FS in 1975 to eliminate the paradox of type-1 fuzzy systems which can be formulized as the problem that the membership grades are themselves precise real numbers [3]. It is not a serious problem for many applications, but, for example, in the case when the features in a pattern recognition application or signal processing have statistical attributes that are non-stationary and the mathematical description of the non-stationarity is unknown, a new approach is neded.

The solution for this problem can be type-2 fuzziness, where fuzzy sets have grades of membership that are themselves fuzzy [4]. At each value of the primary variable *x* on the universe *X*, the membership is a function, and not just a point value (characteristic value). It is the second level, or secondary membership function, whose domain is the primary membership value set. The secondary membership function is a function $MF2:[0,1]\rightarrow[0,1]$. It can be concluded that *MF*2 gives a type-2 fuzzy set which is three-dimensional, and the third dimension design degree somehow the freedom for handling uncertainties.

In [4] Mendel defines and differentiates two types of uncertainties, random and linguistic. The first one is characteristic for example in statistical signal processing, and the linguistic uncertainties characteristic have in word-information based imprecision systems. Operations on type-2 fuzzy sets are extended based on type-1 union, intersection, complementation and usually apply t-norms and conorms. In the process of T2 fuzzy logic inference the uncertainties of fuzzy membership is included in the calculation. In [5] a short review of basic type-2 terms and some special possibilities related to the distance-base operator (DBO) application are given.

## 2.1 Type-2 Fuzzy Sets

A type-1 fuzzy set (T1 FS) has grade of membership that is crisp, whereas a type-2 fuzzy set (T2 FS) has grade of membership that are fuzzy, so T2 FS are 'fuzzy-fuzzy' sets.

One way to represent fuzzy membership of fuzzy sets is to use footprint of uncertainty (FOU), which is a 2-D representation, with the uncertainty about left end point of the left side of the membership function, and with the uncertainty about right end point of the right side of the membership function.

The second way is to use 3D representation, where in the domain *xOy* the F1 FS $A(x)$ is represented, and in the third dimension for every crisp membership value $A(x)$ of the basic variable $x$ a value of possibility (or uncertainty) is given as the function $MF(x,A(x))=\mu(x,A(x))$ . It is the embedded 3D T2 FS (Fig. 1). On Fig. 1 the value $\mu(x,A(x))$ is a random value from the interval [0,1].

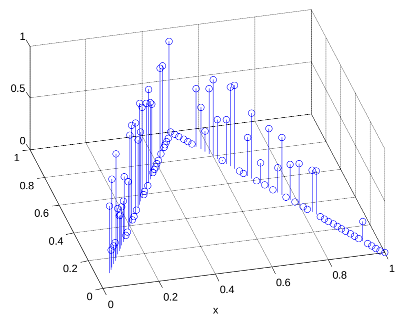


Figure 1
Embedded 3D T2 FS

## 2.2 T2 Result on T2 Operands

Let be $T2A(x, A(x))$ and $T2B(x, B(x))$ T2 fuzzy sets, like at the figure Fig. 1. Let us find $\max_e^{\min}(T2A, T2B)$, calculating it on the domain $\max_e^{\min}(A(x), B(x))$, taking into the account the fuzzy values *T2A* and *T2B* of the fuzzy membership functions $A(x)$ and $B(x)$ [5]. Let the parameter $e$ of the operator $\max_e^{\min}$ fix now.

The value of uncertainly of result is shown on the Fig. 2, and the program solution is as follows:

```
if abs(A(x)-e)<abs(B(x)-e) maxeAB(x)=A(x)
mexe(x)=T2A(x);
else maxeAB(x)=B(x)
maxe(x)=T2B(x)
end,
```

where maxeAB(*x*) is the T1 FS,as the result of the operation $\max_e^{\min}\left(A(x),B(x)\right)$, and the related T2 FS value is maxe(*x*).

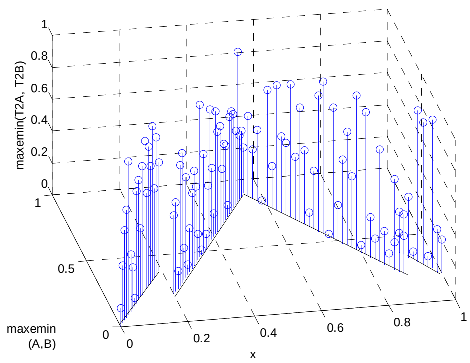


Figure 2
T2 result on T2 FS

## 3 Stochastic Additive ‘Analog to Digital’ Converter

In classical measurement in the whole measurement range absolute error is, more or less, is the same. This is characteristic analog to digital converter. On the other hand, in case of control we know that all measurement subintervals are not of equal signification. This means that equal precision is not necessary for control applications. Fuzzy systems serve as good mathematical models for this simple situation.

Fuzzyfication can be accepted as a kind of measurement, since performing fuzzyfication we confirm membership functions, i.e. that fuzzyfied quantity has characteristics of defined fuzzy sets, depending on the moment - the time variable.

The process of choosing this most significant information is a kind of control process. Human have capability to do this.

Adaptive measurement system can put the process of choosing as most significant information in the control process. It is a human characteristic in the area of automatic control systems to. Let us imagine that we have system with many inputs, one multipexer and one system for processing inputs. On input we have information with low resolution, defined with some synchronizing pulse, but using longer processing time we get information with higher resolution. If processing system works longer on specific input the information is more detailed and more reliable, in the opposite case information is rough and less reliable.

Processing cycle for all inputs lasts the same time, in every case, but work on specific inputs can be variable, depend on system state. So, there is a need for instrument capable to give rough and fast measurement information in the shorter time or precise and reliable measurement information in longer time interval. One of such instruments is stochastic additive A/D converter, introduced in [1].

## 3.1 Function-based Measurements and the Numeric Processing of the Measurements

Stochastic additive A/D converter basically works with equidistant comparators, but this is not obligatory. If we make A/D converter with non-equidistant threshold levels, then we have stochastic additive analog to fuzzy converter (SAAFC).

Defining the membership functions on the output of the stochastic additive analog to fuzzy converter the measurement range is divided in to *n* intervals. There is one deciding threshold in every interval. (*n* +1) sets are defined by *n* deciding thresholds. The first set is approximately the minimum value of the measurement range, $(n+1)^{st}$ set is approximately the maximum value of the measurement range. Between these two sets there are (*n*-1) sets 'approximately $A_j$' where $x_{A_j}$ represents the middle of the two deciding thresholds: $PO_j$ and $PO_{j-1}$ where $j$ = 2,...,*n*. The deciding thresholds are marked from $PO_1$ to $PO_n$. There exists the following limitation: $PO_1 > PO_{j-1}$ where $j = 2,...,n$.

In general case, the membership of the defined sets can be described in the following way:

$$A_j(x) = \{x_i | x_i \geq PO_{j-1} \wedge x_i < PO_j\} \text{ where } j = 2,...,n \quad (1)$$

For the first and the last set:

$$A_1(x) = \{x_i | x_i < PO_1\} \quad (2)$$

$$A_{n+1}(x) = \{x_i | x_i \geq PO_n\} \quad (3)$$

The fuzzy intervals or fuzzy numbers are defined on these sets.

Membership function of fuzzy sets *($A_j$)* is defined depending on the relative frequency of appearing of the measuring result in *($A_j$)* during the measuring cycle.

$$\mu_{A_j}(x) = \frac{a_j}{N} \quad (4)$$

where: $j$ is the serial number of the fuzzy set, $j = 1, 2,...,(n+ 1)$; $a_j$ is the number of appearing value on the output of the fuzzy set $A_j$,;$N$ is the total quantification number during the measuring cycle.

The following limitation is in force for $h$: $h \leq \min(PO_j - PO_{j-1})$, where $j=2,...,n$, where $h$ must be smaller or equal to the minimum difference between the two neighboring deciding thresholds.

The elements of continuous set are ordered to the membership function values as a secondary fuzzy set, constructed a type-2 fuzzy environment (see Fig. 1).

The appearance of the signal in the first instant of quantization indicates rough estimation of specific input and it can define processing time interval. If estimation tells that input is under control, we can process immediately the next input. But if estimation tells that input is not under control, system pay more attention (longer processing time) to go back under control.

If we deal with system with great number of inputs and with limited capability, not only for measurements, but for processing as well, we can use this system more efficiently if we apply above mentioned idea: if processing system works longer on specific input the information is more detailed and more reliable, in the opposite case information is rough and less reliable. Similarly to human reasoning, system can concentrate on most significant input and most significant information is processed. For other inputs, system only confirms that they are under control. For the complex computation of the system states and behaviors in the systems with great number of inputs the distance based operator group is rcommendet.

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## Conclusions

Introduced applications on T2 FS, applying distance based operators are first of all preliminary works to construct inference mechanism based on this family of fuzzy operators and T2 fuzzy logic. Further, in the process is application related to the signal processing. The developed measure method SAADK in [1], stochastic adding A/D conversion has several advantages compared to standard digital instrumentation. Its main advantages are extremely simple hardware and, consequently, simple implementation of parallel measurements, and on the other hand possibility to trade speed for accuracy. This instrument can be either fast and less accurate, or slow and very accurate, depending on the choice of frequency of reading its output, therefore the membership on the universe is time-depending and with random uncertainties, it means suitable to represent it as T2 FS. The hardware realization of the problem is based only on the min and max operators, therefore distance based operators, which are defined using min and max operators will be introduced in the process of realisation. It is an opportunity to use simply hardware realization of the operators and approximate reasoning process in the control problems based on the SAADK method.

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