Definitions and Measurement of Power Factor

András Fehér, dr. Zoltán Puklus

Széchenyi István University,
Egyetem tér 1, H-9026 Győr, Hungary

Abstract: The methodology of measurement of cos(φ) was developed long time ago, but it is usable only for sinusoidal voltages and currents. Nowadays the customer loads use power supplies built with semiconductors, switching power supplies, power controlling systems, etc. These loads cause non-sinusoidal current in power lines, energy quality and EMC problems. The method of Power Factor measurement is described in this article, which is a good KPI in these cases.

Keywords: AC-AC converters, EMC, Measurement, Power factor

1 Intoduction

An AC system of Active power (P), Reactive power (Q), and Apparent power (S) plays a major role in electric power technology. The terms of Active power, Reactive power, and Apparent power are applied to steady-state alternating current circuits in which the voltages and currents are non-sinusoidal. Today it is characteristic in most parts of the applications that the current and voltage are non-sinusoidal.

The origin of these problems are the following

- Nonlinear loads (Single-phase and Tri-phase rectifiers, Cycloconverters, Dimmers, Inductive ballasts for lamps, Arc furnaces, etc.)

- The low power single-phase equipments with rectifier at the front end, from which a large population is operating at the same time (TV, Computer, Electronic ballast, etc.) have the highest impact on the line-voltage

- Important part of distortion is caused by the high-power equipments with three-phase rectifier at the front end (Motor control, On-line UPS, Charger for large batteries, etc.)

Practically during technical descriptions we use only sinusoidal current/voltage components which is obtained by Fourier analysis of shapes of the instantaneous characteristics. There are harmonics.

The effects caused by the line harmonics

Distorted line current

- Reduced power factor means reduced available power, increased distribution losses (exacerbated by sky and proximity effects)

- The third harmonics means excessive current in neutral conductor (danger of fire)

- Overheating of transformers and generators (copper losses)

- Increased audio noise, increased telephone interference

Distorted Line voltage (caused by distorted line current due to the presence of distribution impedance)

- Overheating of transformers and generators (increased iron losses)

- Cogging (refusal to start smoothly) and crawling (very high slip) in induction motors; mechanical oscillations

- Overheating of Power Factor correcting capacitors (premature failure)

- Dielectric stress in insulation systems

- Resonances (overvoltage insulation failure)

- Impaired performance of power-line communication systems

2 Definition of Power Factor

In the earlier definition the power factor is the classical definition, for pure sine-wave current and voltage:

$$PF = \cos(\phi) = \frac{P}{V_{RMS(\sin)} \cdot I_{RMS(\sin)}}$$

The extended definition of Power Factor for non-sinusoidal current and voltage is the following

$$PF = \frac{P}{V_{RMS} \cdot I_{RMS}}$$

Definition for non-sinusoidal current and sinusoidal voltage of power factor is

$$PF = \frac{V_{RMS(\sin)} \cdot I_{RMS(1)}}{V_{RMS(\sin)} \cdot I_{RMS}} \cos(\phi) = \frac{I_{RMS(1)}}{I_{RMS}} \cos(\phi) = K_p K_d$$

where

P	average power (average over a period)
$V_{RMS(\sin)}$	rms voltage (=Vp/sqrt(2))
$I_{RMS(\sin)}$	rms current (=Ip/sqrt(2))
φ	phase angle between the voltage and current
V_{RMS}	rms voltage ($\neq$Vp/sqrt(2))
I_{RMS}	rms current ($\neq$Ip/sqrt(2))
$V_{RMS} I_{RMS}$	apparent power
$I_{RMS(1)}$	rms value of the fundamental harmonic current
K_p	purity (or distortion) factor
$K_d = \cos(\varphi)$	displacement factor

Today it is more important to determine the relation between the Total Harmonic Distortion and Power Factor.

The definition of the Total Harmonic Distortion is

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1}.$$

The relationship between the Total Harmonic Distortion and Distortion Factor can be derived from the following expressions

$$THD = \sqrt{\frac{1}{K_p^2} - 1} \qquad\qquad K_p = \frac{1}{\sqrt{1 + THD^2}}$$

The formula for the Power Factor by means of the Total Harmonic Distortion with zero phase shift between the voltage and the fundamental component of the current is given by

$$PF = \frac{1}{\sqrt{1 + THD^2}}$$

For simple circuits the value of PF and THD can be determined only by computational methods.

3 Circuit Examination

An example circuit diagram of three-phase full-wave controller is shown in the Fig. 1 with wye-connected resistive load.
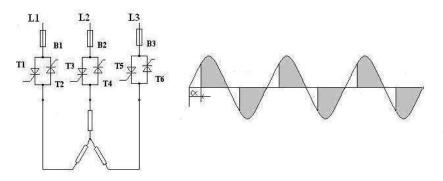

Figure 1

Tree-phase full-wave controller

The shapes of the voltages of the loads are very complicated. The exact shapes depend on the commutation angle (α). In this case one has to use the definition of PF for non-sinusoidal current and sinusoidal voltage of power factor, which is

$$PF = \frac{I_{RMS(1)}}{I_{RMS}} \cos(\phi) = K_p K_d$$

The current in this case is decomposed to its Fourier components according to

$$i(t) = i_1(t) + \sum_{h \neq 1} i_h(t)$$

or

$$i(t) = \sqrt{2} I_1 \sin(\omega t - \varphi_1) + \sum_{h \neq 1} \sqrt{2} I_h \sin(\omega_h t - \varphi_h)$$

$$0 \leq \alpha < \pi/3$$

$$U_{aRMS} = U\sqrt{1 - \frac{3\alpha}{2\pi} + \frac{3}{4\pi}\sin 2\alpha}$$

$$\pi/3 \leq \alpha < \pi/2$$

$$U_{aRMS} = U\sqrt{\frac{1}{2} + \frac{9}{8\pi}\sin 2\alpha + \frac{3\sqrt{3}}{8\pi}\cos 2\alpha}$$

$$\pi/2 \leq \alpha < 5\pi/6$$

$$U_{aRMS} = U\sqrt{\frac{5}{4} - \frac{3\alpha}{8\pi} + \frac{3}{8\pi}\sin 2\alpha + \frac{3\sqrt{3}}{8\alpha}\cos 2\alpha}$$

$$\alpha > 5\pi/6$$
$$U_{aRMS} = 0$$

Figure 2
The RMS as function of α

The active power is calculated by

$$P = UI_1 \cos\varphi_1$$

the purity (or distortion) factor is

$$K_p = \frac{I_{RMS(1)}}{I_{RMS}}$$

whereas the Power Factor is

$$PF = \frac{I_{RMS(1)}}{I_{RMS}}\cos(\phi) = K_p K_d$$

The shapes of voltages (and currents witch is the same) of the loads are very complicated as it can be seen in Fig. 2. To obtain the exact value of the Power Factor, the current spectrum, the I_{RMS} and the $arc(I_1/V_{L1})$, as a function of α the Agilent VEE program was used.
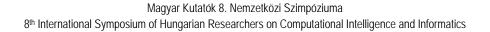
Fig. 3 shows the structure of the model used for computation.

Figure 3

The structure of the program in VEE

This program uses discrete time simulation of the current in 4096 points per period. The studied currents have to be given as mathematical formulas. Disadvantage of these circuits is that even the values of the higher harmonics depend on α.

In Fig. 4 the shape of current at α=135° can be seen, while Fig. 5 shows the level of the harmonics up to 2,5 kHz using a linear scale.
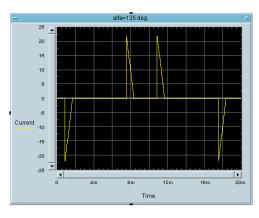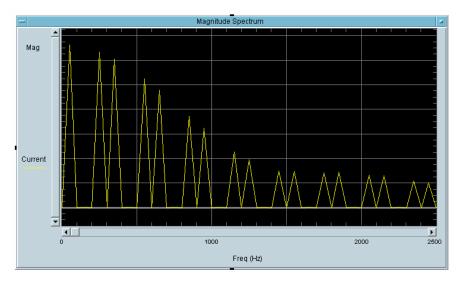
Figure 4
Shape of current (α=135°)

Figure 5
Magnitude of current spectrum (α=135°)

We have calculated the fundamental harmonic of the current (amplitude, phase), the phase angle between the input voltage and the fundamental harmonic of the current, and determined the Power Factor dependence of α. The results are shown in Fig. 6.
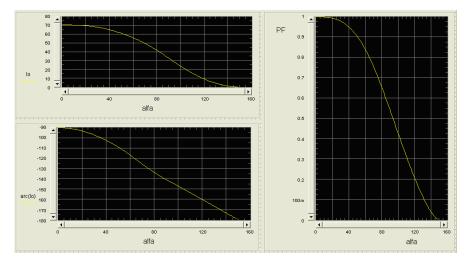
Figure 6
The results of simulation by VEE program

In order to make True Value Measuring Equipment we need to realize the methods sown in Fig. 5. The nth harmonic, the I[n] U[n] outputs of the equipment in Fig. 7, are usable for examination of the harmonics according to the standards of EMC.
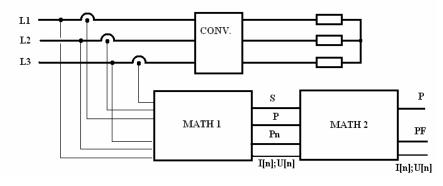

Figure 7
Block diagram of True Measurement

4   Theoretical Consideration

In the circuit we measured the instantaneous values of voltage and currents are noted as (u(t);i(t)). In this case the Active Power (P) is

$$P = \frac{1}{T}\int_0^T u(t)i(t)dt$$

In the sinusoidal case, the shape of Active Power curve is shown in Fig. 8 for firing angles φ=30° and φ=60°. For φ=90° the average of the Active Power is zero.
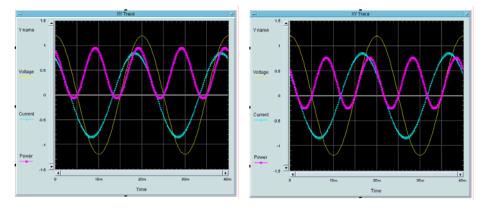

Figure 8
Active Power curve for delays φ=30° and φ=60°

The Apparent Power (S) is defined by the product of RMS values of the voltage and the current,

$$S = UI,$$

the Negative Power, i.e., the feedback to the line, where the instantaneous P is less then zero is calculated by Tröger forms [2]

$$P_n = \frac{1}{2T}\int_0^T \left[|u(t)i(t)| - u(t)i(t)\right]dt,$$

thus the Flowing Power to the load

$$P_f = \frac{1}{T}\int_0^T |u(t)i(t)|dt = P + 2P_n$$

and the positive sign power for the load

$$P_p = P + P_n.$$

We suggest to apply the following formalism. Let *N* be the number of samples in one period *T*, thus the numerically calculated quantities

$$M^+ = \frac{1}{N} \sum_{n=0}^{N} u_n \cdot i_n \bigg|_{\text{if } u_n \cdot i_n \geq 0} \quad \text{and}$$

$$M^- = \frac{1}{N} \sum_{n=0}^{N} u_n \cdot i_n \bigg|_{\text{if } u_n \cdot i_n < 0}$$

can be used for determining the value of *P* as

$$P = M^+ - M^- .$$

Conclusions

The quality of electrical energy is not only a technical, but an economical question, too. Before permeation of semiconductors, the topmost quality measurement of consumers was the $\cos(\varphi)$, and it was easy to measure in sinusoidal case.

Nowadays almost every electronic loads contain power supplies, power controlling systems or semiconductors. In this case the shape of the current is not sinusoidal, therefore the simple $\cos(\varphi)$ can not be a key performance indicator, moreover, it is difficult to be assigned.

In non-sinusoidal current or voltage cases a good consumer load key performance indicator is the Power Factor, but assigning the PF is an even more complicated task.

Acknowledgement

References

[1]   Agilent VEE (Visual Engineering Enviroment) ver. 7.03

[2]   Tröger, R.: Energetische Darstellung von Blindstromvorgängen. ETZ-A Nr. 18 (1953) 533-537

[3]   M. H. Rashid: Power Electronics Prentice Hall 1988

[4]   Mohan – Underlan – Robins: Power Electronics John Wiley & Sons 2003