Improvement of a Fixed Point Transformations and SVD-based Adaptive Controller

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Abstract: In this paper some refinement of a novel control approach is reported that fits to the "traditional line of thinking" according to which in the most practical cases neither very precise, nor even complete system model is needed for obtaining precise control for dynamical systems. The validity of this statement is briefly pointed out in the most popular approaches as the main idea of the "Robust Sliding Mode / Variable Structure Controllers", in the Adaptive Inverse Dynamics and in the Slotine-Li Adaptive Controllers based on Lyapunov's 2nd Method, and in a recently published problem tackling using the simple geometric interpretation of the Singular Value Decomposition (SVD). In the present approach the originally proposed convergent, iterative Cauchy sequences are nonlinearly moderated to adaptively control a coupled nonlinear system, the cart plus double pendulum serving as popular paradigm of dynamicall not very well conditioned systems. It is shown that the proposed moderation removes the small sharp fluctuation in the control torque that inherently belonged to the original solution without significantly degrading the control quality. This statement is substantiated by simulation results.

Keywords: nonlinear control; iterative Cauchy sequences; tangent hyperbolc fixed point transformations; singular value decomposition;

1 Introduction: Precise Dynamic Control without Precise Dynamic Model

The idea of making precise control without being in the possession of any precise or even complete system model has long and great traditions. The most popular idea is this line is the robust "Variable Structure / Sliding Mode Controllers (VS/SM)" originates from Russia from the sixties of the past century and survived in many variants in the forthcoming decades, too (e.g. [1], [2], [3], [4]). It is based on the introduction of the operator $(d/dt+\Lambda)^{m-1}$ that is applied to the trajectory tracking error if the order of the set of differential equations determining the state propagation is *m* [5]:

$$S := \left(\frac{d}{dt}+\Lambda\right)^{m-1} \underbrace{\left(x^{Nom}(t)-x(t)\right)}_{e(t)} \tag{1}$$

In (1) $\Lambda>0$. If $S=0$ then $(d/dt+\Lambda)^{m-2}e(t)\to 0$ exponentially. Roughly speaking it can be stated that during the time $2/\Lambda$ this quantity practically becomes 0. When this situation is achieved the term $(d/dt+\Lambda)^{m-3}e(t)\to 0$ exponentially, etc. Via following this argumentation it can be expected that after finite time the tracking error $e(t)$ starts to converge to zero exponentially. Since by calculating the time-derivative of *S* in (1) the desired m^{th} order time-derivative $x^{(m)Des}$ can be determined, in the typical case of robust controllers an approximate system model used to be satisfactory to drive *S* into the vicinity of 0 during finite time. (Subtle details of this convergence do not have practical significance.) For this purpose various strategies can be described. A typical choice used to be the prescription of the desired time-derivative $\dot{S}^{Des}$ as

$$\dot{S}^{Des} = -K\,\mathrm{sgn}(S) \tag{2}$$

with satisfactorily big positive constant *K*. The phenomenon of chattering used to occur due to abrupt jumps in the sign of *S* in (2). A possibility for reducing chattering is smoothing the variation of $\dot{S}^{Des}$ in (2) is the introduction of a switching layer within which thid value varies continuously without abrupt jumps. This smoothing normally reduces the precision of trajectory tracking. Normally (2) can be used for calculating the desired derivative $x^{(m)Des}$ that can well be approximated by the available rough model of the dynamics of the system.

Lyapunov's 2nd Method that also used to be applied in connection with the VS/SM controllers is even a far more older idea. In his PhD Thesis Lyapunov originally investigated the stability of dynamic systems [6]. Besides the fact that such systems normally provide equatins of motion that cannot be solved (integrated) in closed analytical form, about the end of the 19^{th} Century, in the lack of computers of useful computational power, it was also hopeless to numerically solve them in order to investigate the behavior of their solutions. Lyapunov's greatness stands in the fact that he was able to give definite statment on the stability of the equilibrium points of such systems *only on the basis of simple estimations* for which knowing the detailed behavior of the solutions "kicked out of the equilibrium point" was quite unnecessary. Since systems controlled with feedback loops behave like closed dynamical systems in which the equilibrium points correspond to zero tracking error, Lyapunov's work was also issued later (e.g. [7]), and became the basis of designing various adaptive controllers (e.g. [8]. It is worth noting that as in the case of the VS/SM controllers, the details of the error relaxation remains unknown and practically unimportant in the case of the sophisticated dynamic controllers developed on the basis of Lyapunov's idea as e.g. in the cases of the *Adaptive Inverse Dynamics* or the *Adaptive Slotine - Li* controllers. Actually these controllers operate on the basis of an approximate system model while continuously "learning" the excerpts of the "exact" one. The excellent trick applied in the Sotine-Li approach that deserves especial attention is that instead of introducing a more or less "arbitrary" positive definite matrix for constructing the Lyapunov function, the exact symmetric positive definite inertia matrix of the mechanical system is used for constructing it. *It is very important to realize that the unknown this matrix itself is not used in calculating the control signal. Only its existence and properties are used in the estimation that guarantees non-positive time-derivative of the Lyapunov function.* It is worth noting that for tuning the parameters normally the necessary information is present in various sections of the trajectory of the controlled system, so this learning generally is not a monotone process.

Since in the use of the above mentioned adaptive dynamic approaches *it is assumed that the generalized forces calculated by the controllers are the only contributions and no additional external perturbations are present*, these sophisticated approaches have a kind of "weak point". Unknown external perturbations can fob their operation. Another probem with the techniques using Lyapunov's 2*nd* Method is that normally it is not very easy to find an appropriate Lyapunov function candidate. It is rather an art than a simple practice. Furthermore, the structure of the Lyapunov function chosen seriously restricts the nature of the applicable feedback that limits the quality of the so obtained control. This aspect was investigated in details in [9] in comparison with the operation of a novel problem tackling based on the simple geometric interpretation of the *Singular Value Decomposition (SVD)* for *Multiple Input-Multiple Output (MIMO)* systems. In this control the SVD meaning relatively great computational burden (e.g. [10, [11]) has to be executed on a very approximate model outside of the control cycle only in certain points of the configurational space. The results in these "grid points" can nonlinearly be interpolated within the control cycle as in the case of the *Support Vector Machines (SVM)* using *Radial Basis Functions (RBF)*. The method of *Higher Order Sinular Value Decomposition (HOSVD)* recently became a fundamental tool for developing models of uniform structures for various physical systems [12]. In the background of this method the existence of the cheap but considerable computational power of the common personal computers of our days can be recognized that is applicable offline for constructing the model. Due to the fact that using simple steps of lucid geometric interpretation is far easier than inventing an appropriate Lyapunov function, in the sequel an attempt based on this idea will be presented.

2 The Excitation - Response Scheme and Fixed Point Transformations

Each control task can be formulated by using the concepts of the appropriate ”excitation” **Q** of the controlled system to which it is expected to respond by some prescribed or “desired response" $\mathbf{r}^d$. The appropriate excitation can be computed by the use of some inverse dynamic model $\mathbf{Q}=\varphi\left(\mathbf{r}^d\right)$. Since normally this inverse model is neither complete nor exact, the actual response determined by the system's dynamics, ψ, results in a *realized response* $\mathbf{r}^r$ that differs from the desired one: $\mathbf{r}^r=\phi\left(\varphi\left(\mathbf{r}^d\right)\right):=\mathbf{f}\left(\mathbf{r}^d\right)$. It is worth noting that these functions may contain various hidden parameters that partly correspond to the dynamic model of the system, and partly pertain to unknown external dynamic forces acting on it. Due to phenomenological reasons the controller can manipulate or “deform” the input value from $\mathbf{r}^d$ so that $\mathbf{r}^d=\mathbf{f}\left(\mathbf{r}_*^d\right)$. Other possibility is the manipulation of the output of the rough model as $\mathbf{r}^d=\phi\left(\varphi_*\left(\mathbf{r}^d\right)\right)$. In the sequel it will be shown that for *SISO* systems the appropriate deformation can be defined as some *Parametric Fixed Point Transformation*. The first efforts in the direction of applying uniform structures and procedures in quite different way as it is done in the classic *Soft Computing* applications were summarized in [13] in which the sizes of the necessary uniform structures used for developing *partial*, *temporal*, and *situation-dependent* models that needed continuous maintaining were definitely determined by the degree of freedom of the system to be controlled. These considerations were based on the modification of the *Renormalization Transformation*, and were valid only for “*increasing systems*” in which the “increase” in the necessary response could be achieved by also increasing the necessary excitation, and *vice versa*. In [14] this idea was systematically extended for *Single Input - Single Output (SISO)* “increasing” and “decreasing” systems by developing various Parametric Fixed Point Transformations more or less akin to the Renormalization Transformation. The latest version elaboarted for SISO systems was the function

$$G\left(x;x^d\right)=\left(x+K\right)\left[1+B\tanh\left(A\left[f\left(x\right)-x^d\right]\right)\right]-K \qquad (3)$$

ith the following properties: if $f(x_*)=x^d$ then $G(x_*,x^d)=x_*$, $G(-K,x^d)=-K$, and

$$G' = (x+K)\frac{BAf'(x)}{\cosh^2\left(A\left[f(x)-x^d\right]\right)} + \left[1 + B\tanh\left(A\left[f(x)-x^d\right]\right)\right] \tag{4}$$

that can be made contractive in the vicinty of x_* by properly setting the parameters A, B, and K, in which case the iterative sequence $x_{n+1}=G(x_n,x^d)\rightarrow x_*$ as $n\rightarrow\infty$. The saturated nonlinear behavior of the tanh function played very important role in (3).

The latest constructio proposed for MIMO systems considered the $\mathbf{x}^d=\mathbf{f}(\mathbf{x}_*)$ task of generating small correction of the output f towards the desired value by applying small corrections for smooth functions with a small positive α factor as

$$\Delta\mathbf{f} = \frac{\partial\mathbf{f}}{\partial\mathbf{x}}\Delta\mathbf{x} = \alpha\left(\mathbf{x}^d - \mathbf{f}(\mathbf{x})\right) \tag{5}$$

If the Jacobian of **f** can be inverted then the following iterative sequence of points can be generated by this approach:

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \alpha\left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]^{-1}\left(\mathbf{x}^d - \mathbf{f}(\mathbf{x}_n)\right) \tag{6}$$

Really, by estimating the error of the next step a Cauchy sequence of decreasing errors can be revealed by this method since

$$\begin{aligned}\mathbf{x}^d - \mathbf{f}(\mathbf{x}_{n+1}) \approx\approx \mathbf{x}^d - \mathbf{f}\left[\mathbf{x}_n + \alpha\left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]^{-1}\left(\mathbf{x}^d - \mathbf{f}(\mathbf{x}_n)\right)\right] \approx \\ \approx \mathbf{x}^d - \mathbf{f}(\mathbf{x}_n) - \alpha\left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]\left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]^{-1}\left[\mathbf{x}^d - \mathbf{f}(\mathbf{x}_n)\right] \approx (1-\alpha)\left[\mathbf{x}^d - \mathbf{f}(\mathbf{x}_n)\right]\end{aligned} \tag{7}$$

since $0<(1-\alpha)<1$. If the Jacobian is not precisely known the error still can be made decreasing if the angle between the “ideal” and the “realized” displacements in this psece is acute. For rough calculation the SVD for an available approximate Jacobian as $\partial\mathbf{f}/\partial\mathbf{x}=\mathbf{UDV}^T$, $[\partial\mathbf{f}/\partial\mathbf{x}]^{-1}=\mathbf{VD}^{-1}\mathbf{U}^T$ can be used that, in the above outlined iteration, leads to the step $\Delta\mathbf{x}=\mathbf{VD}^{-1}\mathbf{U}^T[\mathbf{x}^d-\mathbf{f}(\mathbf{x})]$. Taking into account that in the result of the SVD the singular values appear in an ordered sequence, and that the orthogonal unit vector columns of the orthogonal matrices can serve ad orthonormal basis vectors, by the use of the associativity of the matrix product, the desired step can be estimaed as

$$\begin{aligned}\mathbf{x}^d - \mathbf{f}(\mathbf{x}) = \sum_l c_l\mathbf{u}^{(l)},\ c_l = \left(\mathbf{u}^{(l)}, \mathbf{x}^d - \mathbf{f}(\mathbf{x})\right) \\ \Delta\mathbf{x} \approx \alpha\sum_l D_{ll}^{-1}c_l\mathbf{v}^{(l)}\end{aligned}. \tag{8}$$

If we have some quantitative order of magnitude values for the appropriate spaces L, the proper value of α can be estimated as follows: $\alpha \approx LD_{nn}\Big/\left(\sqrt{n}\max_{l=1}^{n}|c_l|\right)$

(too small α leads to too slow convergence and low quality control, while too big α may lead to divergence. It is important to note that it is not needed to execute the operation of SVD within the control cycle. It can be done previously by calculating the proper matrices in certain grid points of the configurational space, and following that a kind of nonlinear interpolation as also applied e.g. in the case of Support Vector Machines can computationally very efficiently used within the control for estimating these matrices. This approach worked well for the adaptive control of the cart plus double pendulum system. However, it had little deficiencies as folows: the actual estimation of α always fluctuated and remained some finite value that also caused small fluctuations in the control signal when the error already was in the vicinity of zero.

To remove this fluctuation in the present approach the following modifications were done: in each control step a limited estimation was applied to α as $\alpha_{Est} \approx LD_{nn} \Big/ \left(\sqrt{n}\max\left[\max_{l=1}^{n}|c_l|,1\right]\right)$ in a memory variable the so obtained maxial value α_{max} was stored, and the actual value of α in use was $\alpha = \alpha_{\max}\tanh\left(0.5\left\|\mathbf{x}^d - \mathbf{f}\right\|/L\right)$. In this approach for small errors $\alpha \to 0$ to avoid fluctuations, and for big errors it approaches α_{max}. This modification evidently applies a similar refinement that also applied in the case of the SISO systems using a tangent yperbolic form fir the function *G*. In the sequel simulation examples will be given for the cart plus double pendulum system to illustrate the operation of the original and the refined methods.

3 The Dynamic Model of the Paradigm

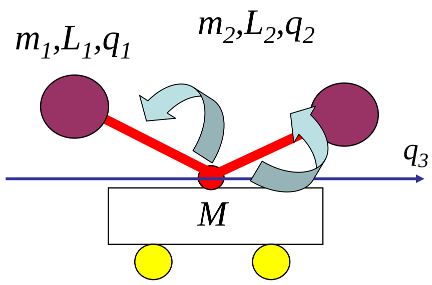


Figure 1
Sketch of the cart plus double pendulum system

The cart + double pendulum system is a typical example for mechanical systems having badly conditioned inertia matrix in the vicinity of certain critical points of the configurational space. Its rough sketch is given in Fig. 1. It consists of a cart serving as a body rolling on wheels of negligible momentum and inertia having the overall mass M, pendulums assembled on the cart by parallel shafts and arms having negligible masses and lengths L_1 and L_2, respectively. At the end of each arm a ball of negligible size and considerable masse m_1 and m_2 are attached. The Euler-Lagrange equations of motion of this system are as follows:

$$\begin{aligned}&[Q_1 \quad Q_3 \quad Q_3]^T = \\ &= \begin{bmatrix} m_1 L_1^2 & 0 & -m_1 L_1 \sin q_1 \\ 0 & m_2 L_2^2 & -m_2 L_2 \sin q_2 \\ -m_1 L_1 \sin q_1 & -m_2 L_2 \sin q_2 & (M + m_1 + m_2) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \\ &+ \begin{bmatrix} -m_1 L_1 \cos q_1 \dot{q}_1 \dot{q}_3 - m_1 g L_1 \cos q_1 \\ -m_2 L_2 \cos q_2 \dot{q}_2 \dot{q}_3 - m_2 g L_2 \cos q_2 \\ -m_1 L_1 \cos q_1 \dot{q}_1^2 - m_2 L_2 \cos q_2 \dot{q}_2^2 \end{bmatrix}\end{aligned} \tag{9}$$

in which g denotes the gravitational acceleration, Q_1 and Q_2 denote the driving torques at the rotary shafts, and Q_3 stands for the force moving the cart in the horizontal direction. The appropriate rotational angles are q_1 and q_2, and the linear degree of freedom belongs to q_3. The determinant of the inertia matrix has the form of

$$\begin{aligned}&\det \mathbf{H} = m_1 L_1^2 m_2 L_2^2 \times \\ &\times \left(M + m_1 + m_2 - m_1 \sin^2 q_1 - m_2 \sin^2 q_2\right)\end{aligned} \tag{10}$$

the minimum value of which is equal to $(\det \mathbf{H})_{\min} = m_1 L_1^2 m_2 L_2^2 M$. The "critical" points belong to the minimum of the determinant of the inertia matrix in the coincidence of the "critical coordinate values" q_1, $q_2=\pm\pi/2$. On this reason in the present, extended paper, the main idea of the RBFNs was used by "spanning the tent-cloth" over the grid points at $\pm\pi$, $\pm\pi/2$, and 0 for both q_1, and q_2 that means 5×5=25 points with the radial function $d_{ij}(q_1, q_2) = \exp\left(-4\left((q_1 - q_{1,ij})^2 + (q_2 - q_{2,ij})^2\right)\right)$. In the estimation of the **U**, **V**, and **D** matrices these d_{ij} values were used for weighting. The SVD was executed only in the grid-points prior to initiating the control. Calculation of such a weighted average of a few small matrices does not mean considerable computational burden. These grid-points do not concern the Slotine-Li control. For describing the phenomenon of friction the Lund-Grenoble model [15, 16] was used which the deformation of the bristles of some "brushes" are applied to describe the deformation of the surfaces in dynamic contact, so friction is described as a dynamic coupling between two systems having their own equations of motion as

$$\begin{aligned}&\frac{dz}{dt} = v - \frac{\sigma_0 |v| z}{F_C + F_S \exp\left(-|v| / v_s\right)}, \\ &F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + F_v v\end{aligned} \tag{11}$$

for which the proper direction of F has to be set in the applications. Variable v denotes the relative velocity of the sliding surfaces, F_v describes the viscous friction coefficient, z denotes the deformation as an internal degree of freedom, σ_0 plays the role of some "spring constant" of the internal deformation, and σ_1 is a new parameter pertaining to the effect of the bending bristles. The F_C, F_S, and v_s parameters' role is the description of sticking. This model evidently yields $dz/dt=0$ for $v=0$ that can result finite friction force at even zero velocities. The behaviour of the whole system is described by the dynamic coupling between the hidden internal and the observed degrees of freedom. Though the appropriate quantities in this model were developed for linear motion and forces, they easily can be generalized for rotary motion in which torques appear in the role of the forces, and rotational velocity is present instead of linear motion's velocity. The model given by the Euler-Lagrange equations evidently can be completed via adding the additional torque of the friction to the appropriate components of **Q** in it. In general it is very difficult to identify the friction parameters. It seems to be more expedient to apply simple adaptive approach that completely evades such identification problems. The here proposed FPT/SVD-based method just corresponds to this idea more or less akin to the idea of "situational control" [17] in the sense that no complete system model has to be built up for control purposes. In the sequel simulation resulst will be presented.

4 Simulation Results

In the present paper the following model inertia and dynamical parameters were used for the controlled system: M=5 *kg*, m_1=6 *kg*, m_2=4 *kg*, L_1=2 *m*, L_2=3 *m*, g=9.1 m/s^2. The friction models had the following parameters: σ_{01}=10 *Nm/rad*, σ_{11}=150 *Nms/rad*, F_{v1}=1 *Nms/rad*, F_{C1}=100 *Nm*, F_{S1}=200 *Nm*, v_{s1}=0.1 *rad/s* for the 1st axle, σ_{02}=20 *Nm/rad*, σ_{12}=300 *Nms/rad*, F_{v2}=2 *Nms/rad*, F_{C2}=200 *Nm*, F_{S2}=400 *Nm*, v_{s2}=0.2 *rad/s* for the 2nd axle, and σ_{03}=30 *N/m*, σ_{13}=450 *Ns/m*, F_{v3}=3 *Ns/m*, F_{C3}=300 *N*, F_{S3}=300 *N*, v_{s3}=0.3 *m/s* for the 3rd axle. For numerical computation simple Euler-integration was used with the time resolution of δt=1 *ms*. In the tests concerning the effects of the imprecision of the dynamic models the roughly approximate multiplicative factors were used: 0.6 for M, , 0.5 for m_1, and 0.4 for m_2.

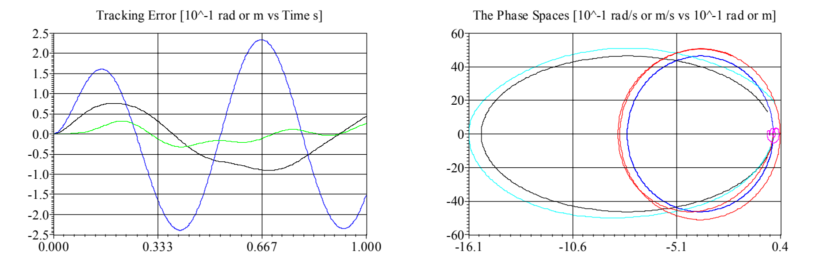
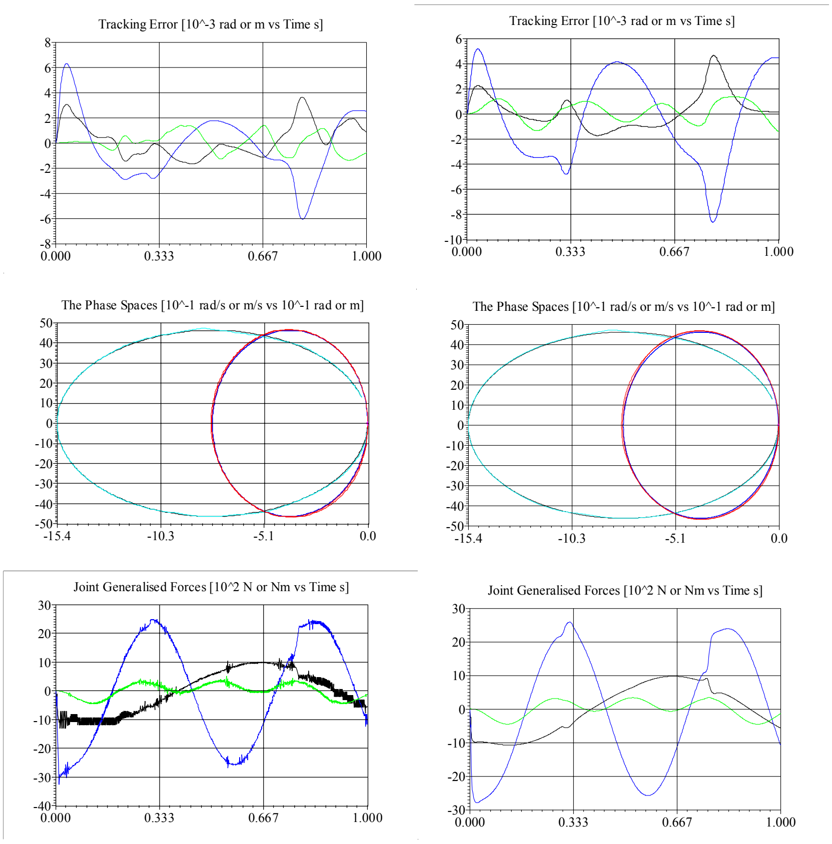


Figure 2

Trajectory tracking error and phase trajectories obtained without adaptivity

As it can be seen in Fig. 2 the rough model and the friction forces lead to low quality control without any adaptivity.



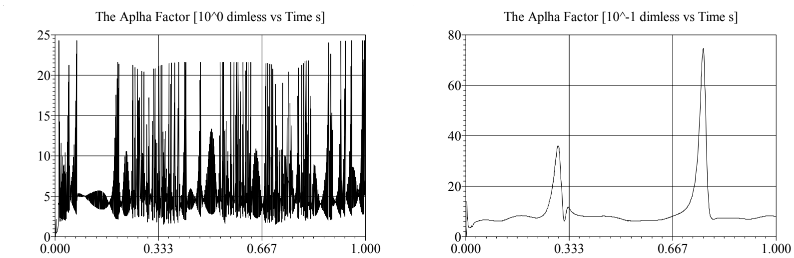


Figure 3
Comparison of the operation of the original (LHS) and the smoothed (RHS) versions of the SVD and fixed point transformations based methods for model inacurracies and without external disturbations

Fig. 3 reveals that both the original and the smoothed control yield good adaptive tracking with comparable generalized forces. However, the latter one yields smooth torque and force signals ans α factors.

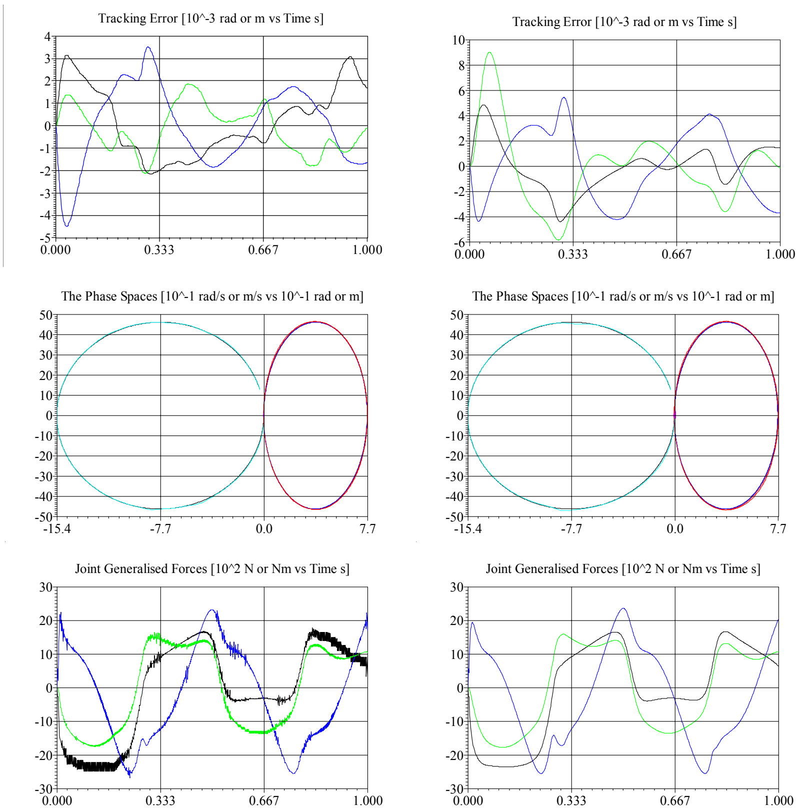


Figure 4
Comparison of the operation of the original (LHS) and the smoothed (RHS) versions of the SVD and fixed point transformations based methods for model inacurracies for strong external disturbation force acting on the cart

According to Fig. 4 similarly good operation cab be obtained for strong external disturbation force actin on the cart.

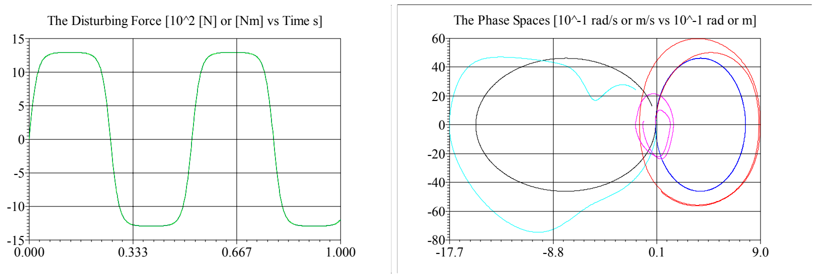


Figure 5

The disturbance force applied on the cart vs time and the phase space of the non-adaptive version

The apropriate disturbing force action on the cart and the phase trajectory of the non-adaptive motion is given in Fig. 5. It can well be seen that besides the effects of the the friction forces that of quite considerable disturbances can also be compensated by the proposed method.

Conclusions

In this paper a nonlinear refinement of a previously proposed SVD-based adaptive nonlinear control for MIMO systems was reported that was developed according to the analogy wit nonlinear fixed point transformations applied for SISO systems. According to the simulations resulst the so obtained control seems to be smooth, reliable, and geometrically simple for interpretation.

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