Geometric Error Correction in Coordinate Measurement

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Abstract: Software compensation of geometric errors in coordinate measuring is hot subject beause it results the descrease of manufturing costs. The papergives a summary of the results and achivements of earlier works on the subject. In order to improve these results a methode is adapted to capture simultaneously the new coordinate frames in order use exact transformation values at discrete points of the measuring volume. The interpolation techniques used have the draw back that they could not maintain the orthogonality of the rotational part of the transformation matrices. The paper gives a technique based quaternions which avoid this problem andleads to better results.

1 Introduction

Three dimensional coordinate metrology is a firmly established technique in industry. Their universal applicability and high degree of automation accounts for it's success in the last 30 years. In order to full-fill its task to verify the geometry of products on the basis of the measured results, CMM-s must be in principally be an order of magnitude more accurate than the machine tool used to manufacture the part. Over the last 50 years one can observe enormous enhancement in positioning and measuring accuracy. The main portion of this enhancement is the result of improved knowledge about high precision machine design [18].

A foundamental principle was recognised by professor Abbe already in the 1890's about the aligment of the displacement measuring system with the distance to be measured. Another fundamental principle is the separation of the structural and measuring functions in a machine. Already in the 1880's measuring equipment was built in wich the measuring system was attached to a separate metrology frame. The third important factor, to be conserned, is the thermal distorsion of the metrology system. A short overview of novel constructions for high precision coordinate measuring machines is given in [8].

As mechanical accuracy is costly, whereas repeatability is not expensive, software techniques were used from the beginning to compensate for the systematic errors in order to keep manufacturing costs low.

One of the eariest paper on error compensation of coordinate measuring machines is by Zhang et al. [25]. They describe the compensation of a bridge type industrial three-coordinate measuring machine, which resulted in an accuracy improvement by approximatly a factor 10. The machine consist of only translational axis and the infinitesimal rotation errors are described by the rotation matrix where the trigonometric functions are replaced by the first term in their Taylor series. The correction vectors are determined at equally spaced points in the measuring volume and are stored in the memory of the computer in the form of look-up table. The correction vectors at intermediate point are calculated simply by linear interpolation.

An analytical quadratic model for the geometric error of a machine tool was developed by Ferreire and Liu [6] using rigid body kinematics. They introduced the notion of shape and joint transform. The former describes the transformation between the coordinate system on the same link and latter the transformation across a joint. To represent the transformations they introduced the use of homogeneous transformation in matrix form. A quadratic expression was developed for the case where the individual joint errors vary linearly with the movement along the joint or axis. The global error description was obtained by concatenating these matrices.

Duffie and Yang [2] invented a methode to generate the kinematic error functions from volumetric error measurements. To represent the displacement error a vectorial approach was followed. The rotational errors were described by matrices in which, taking into account that the angular error are small, the cosine and the sine terms can be approximated with the lowest order terms of their Taylor series. The model was used to describe the error of a measuring probe neglecting the rotation. The translational error components were approximated by cubic polynomials. To find the coefficients least square fit was applied.

Teeuwsen [20] described the error motion of the kinematic components of a coordinate measuring machine by using homogeneous transformations and concatenating these transformations to calculate the resulting global error. Assuming that the rotational errors are very small, he neglected the second order term. Hereby he could ensure the commutativity of the matrices, but at the same time the orthonormality of these matrices was lost, whic means that they do not represent a pure rotation anymore. The error motions of the probe displacements was also handled by tranformation matrices. In order to establish the error map in the form of correction vectors the various error components were measured on a semi automatic way at discrete points of the measuring volume. To obtain a continuous description of the correction vector, between these points, regression was used to establish a piecewise polynomial representation.

Ruijl [15] has build a high precision coordinate measuring machine with a measuring uncertainty of 50 nm in a 100x100x40 mm measuring volume. The machine has a novel contruction, where the air bearing tabel performs the measuring motion in all the three principal directions. It was derived that if the measuring systems are aligned with the centre of the probe tip the relationship between the position of the measuring system and the contact point on the workpiece is unique. This means that the functional point is the centre of the probe tip and hence it is possible to comply with the Abbe priciple. The nano measuring machine of SIOS [17] is based on the same principles. This machine is currently applied as the stage for a long range scanning microscoop.

Kim et al. [12] have constructed an unusual machine. One attempt to get rid of the parallax error of orthogonal type coordinate measuring machines is the application of the so-called multilateration. It is to measure the diagonal distances of the probe using tracking laser interferometers with retro-reflectors. The paper describes a scheme of multilateration based on a single volumetric interferometer system. The volumetric interferometer generates two spherical wavefronts from the probe by using diffraction point sources. The emanetd wavefronts interfere within the measuring volume, while two dimensional array of photodetectors mounted on the machine frame capture the interferometric intensity field. Phase information is used, from which the coordinates of the probe are determined. A second interferometer is installed to measure the x and y position of the machine table.

Kim and Chung [11] also applied infinidesimal matrix transformation to correct the position error due to geometric inperfections and transient thermal error of a machine tool to improve on machine measurement accuracy. Thermal error were derived from the thermal drift of the spindle in the three principle directions.

The static and transient thermal errors and their compensation are discussed by Kruth et al. [13]. Capturing temperature distribution of the machine structure the thermal deformations can be calculated using the linear thermal expansion coefficients of the individual machine components. However the determination of sensor positions in a cumbersome trial and error task.

Recently in a paper Tan and his coauthors [19] describe the application of neural networks for the error compensation of single-axis, a gentry and X-Y stage. The advantage of using neural networks is in the followings: they could be used to approximate any continuous mapping, this mapping can be achived by learning and parallel processing and nonlinear interpolation can be achived. Using this technique the authors could improve the positioning accuracy depending on the configuration investigated by a factor between two and three.

2 Overview of the Errors and their Sources

When considering the mechanical accuracy of coordinate measuring devices three primary sources of quasi-static errors can be identified:

- Geometric errors due to the limited accuracy of the individual machine components such as guideways and measuring systems,
- Errors related to the final stiffness of those components, mainly by moving parts,
- Thermal errors as expansion and bending of guideways due to uniform temperature changes and temperature gradients.

Geometric errors are caused by out of straightness of the guideways, imperfect alignment of the axis and flatness errors.

Deformations in the metrology frame introduce measuring errors. During measurement the deformation of the metrology frame is caused by the probing force. Its effect can be predicted with relatively high precision if the probing force is known and therefore it easily incorporated into the model.

The static deformation of the table is caused be gravity forces. It manifests itself as a contribution to out of flatness error. That means it can be handled on a similar way.

The largest deformations of the metrology frame are thermally induced. The main sources of the thermal disturbance are:

- heating and cooling source in the environment, like lighting, air conditioning, people around the machine, etc.,
- heat generated by the machine itself,
- thermal memory: heat stored in the machine components from a previous thermal state.

The compensation of thermally induced errors is rather cumbersome, because of the complexity of the problem [13]. Based on results from the literature a linear thermal compensation model can be used.

3 Geometric Error Model

A coordinate measuring machine is a multiaxis machine consisting of a chain of translational and/or rotational kinematic components. The geometric deviation of a CMM is originating from the geometric deviations of its components. In order to discuss a general model the error model of the components are dicussed.

A linear stage of precision machinery is expected to travel along a straight line and stop at a predefined position. However in the practiced the actual path deviate from the straight line due to the geometric errors of the guideways and it results also in angular errors as it is given in Fig. 1.

For each axis a transformation matrix can be used to describe in homogenious co-ordinates the deviations from the ideal motion. The general form of a transformation is given by: These components are called guided element. The guided elements are linked by so-called connecting elements, which can be represented by matrices with similar structure with only constant elements. The squarness or perpendicularity error can be represented by a socalled shear matrix of the following form, dependent on which axis is taken as a reference. Here the z axis was taken as a reference.

$$R_{Terr}=\begin{bmatrix} c(\theta_y)c(\theta_z) & -c(\theta_y)s(\theta_z) & s(\theta_y) & \delta_x \\ c(\theta_x)s(\theta_z)+s(\theta_x)s(\theta_y)s(\theta_z) & c(\theta_x)c(\theta_z)-s(\theta_x)s(\theta_y)s(\theta_z) & -s(\theta_x)c(\theta_y) & \delta_y \\ s(\theta_x)s(\theta_z)-c(\theta_x)s(\theta_y)c(\theta_z) & s(\theta_x)c(\theta_z)-c(\theta_x)s(\theta_y)s(\phi_z) & c(\theta_x)c(\theta_y) & \delta_x \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

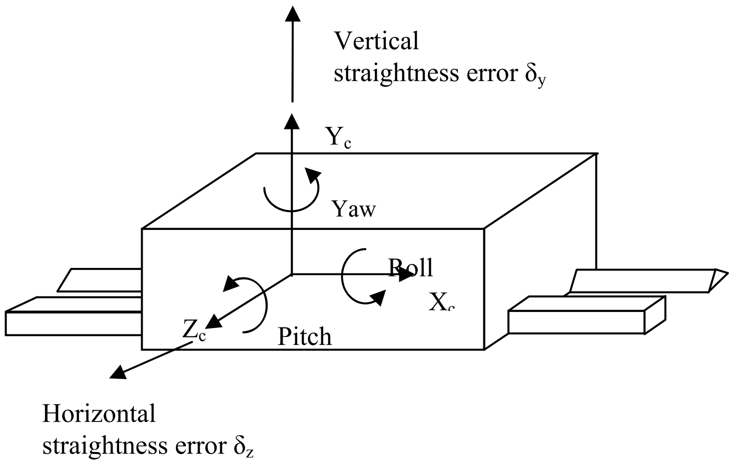

Figure 1
Representation of the six deviations of a translational kinemetic component

Where dx, dy and dz are the translational and *Φ, θ* and *Ψ* are the rotational components and s respectively c are short for sin and cos.

In case of the coordinate table the angular errors are very small, and all the errors are position dependent the following approximation can be made:

$$R_{Terr} = \begin{bmatrix} 1 & -\theta_z(x) & \theta_z(x) & \delta_x(x) \\ \theta_z(x) & 1 & -\theta_x(x) & \delta_y(x) \\ -\theta_y(x) & \theta_x(x) & 1 & \delta_z(x) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Analog results can be derived for the y and z axis. Rotational components can presented on the same way and results in rather similar matrix.

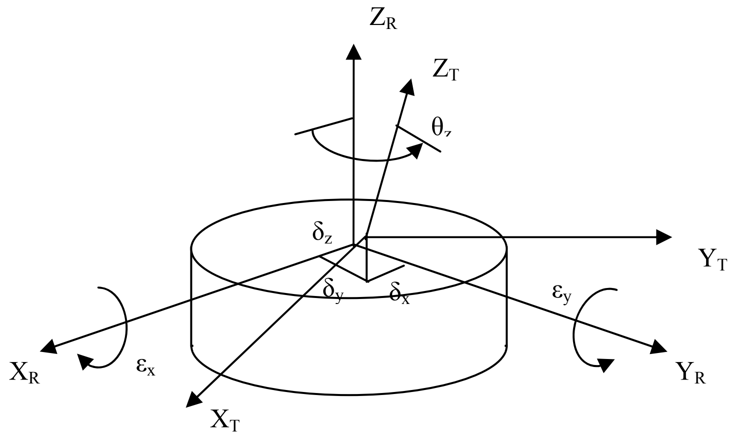

Figure 2
Representation of the six deviation of a rotational kinematic component

These components are called guided element. The guided elements are linked by socalled connecting elements, which can be represented by matrices with similar structure with only constant elements. The squarness or perpendicularity error can be represented by a socalled shear matrix of the following form, dependent on which axis is taken as a reference. Here the z axis was taken as a reference.

$$R_{Perr} = \begin{bmatrix} 1 & 0 & sh_x & 0 \\ 1 & 1 & sh_y & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The resulting error matrix can be obtained by multiplying the individual matrices in the sequence as they follow each other in the kinematic chain.

A traditional co-ordinate measuring machine consists of three translational components x, y and z, and a probe is attached to the end of the z component. Usually the probe can be considered as a constant translational transformation.

Performing distance measurement in the measuring volume of the co-ordinate measuring machine we observe the relative change of the probe tip, therefore only the relative deviations are of interest.

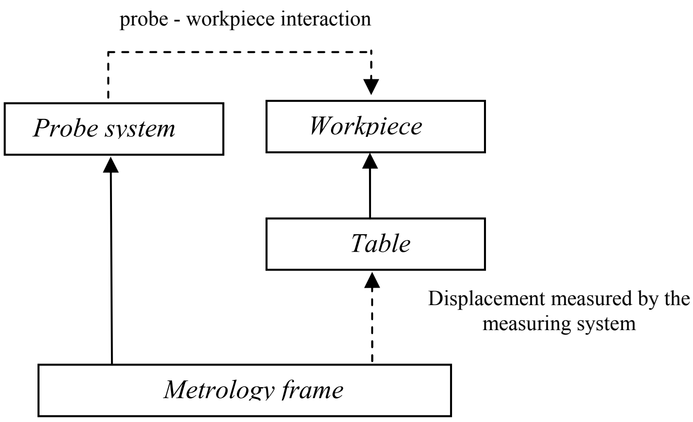

Figure 3
Schema of the metrology loop

In case of a measuring probe its error components can be handled on the similar way as it was done in case of a carriege and a rotattional element.

4 Determination of the Geometric Errors by Measurement

For the calibration of coordinate measuring machines Zhang et al. [23] proposed to determine the angular errors by measuring the displacement errors alog two parallel lines to the axis of motion but separeted by a distance in the appropriate orthogonal direction.

In a more recent paper Zhang and Fu [24] describe the calibration of optical CMM-s using an uncalibrated reversible grid plate in three positions. In the initial position the plate is aligned with the machine coordinate system. Next it si reversed about Y axis of the machine. In the third position the grid is rotated 90º about the Z axis. To determine the scale error one of the machine axis should be calibrated by a laser interferometer.

A simple measuring technique was invented by Fan et al. [3] to determine the motion accuracy of a linear stage. The idea is based on the fact that the position and orientation of a rigid body can be determined by appropriatly selected six point. They measure the displacement of these points and calculate from them the rotational and translational error components. The displacement in the motion direction and the angular errors (pitch and yaw) perpendicular to this directions are measured by three laser interferometers. The roll and the straightness errors are captured by an optical setup containing two quadrant photodetectors. Taking again into considerations that the angular error are small their values are replaced by their tangent. The invention initiated the development a dual and triple beam interferometers.

The above mentioned authors published a paper [5] about the measurement to determine the accuracy of a high precision wafer stage. Therefor 6-DOF errors of its positioning accurcy is significant. An improved version of the above described system was used. The moving part of it is an L-shaped mirror and on top of one leg a long right angle mirror. The stationary part consists of four laser heads, two beam splitters and two quadrant photo detectors. The laser heads use four laser Doppler scales, three of which are parallel to each other. The uper two laser beams can be reflected by the long right-angle mirror and the lower one by the Y leg of the L shaped plane mirror. The fourth laser beam is aligned in the X-axis and reflected by the X mirror. Comparing the four linear measurements by four laser doppler scales, the X and Y positioning error of the moving table and its pitch and yaw errors can be determined. The upper two reflecting beams are split and each split beam is received by a quadrant photo detector. Comparing these signals the vertical straightness and the roll error can be derived at the same time. In order to minimize the cosine errors among the three displacement measurement and to ensure angular accuracy of the pitch and yaw measurements the parallelism of these beam should be precisely adjusted. The system takes into consideration the squareness aligments and the the flatness error of the plane mirrors.

In their paper Gao et al. [7] describe the measurement strightness and rotational error motions of a commercially available linear airbearing stage actuated by a linear motor. The pitch and yaw errors wre measured by an autocllimator. For the roll error measurement two capacitive displacement probes scan the flat surface in the XZ plane The probes with their sensing axis in the Y direction were aligned with a certain spacing. The roll error is obtained by dividing the difference of the outputs of the two probes by the spacing between them. The horizontal and vertical strightness error were measured by using the strigthness kit of a laser interferometer.

The setup to detect motion errors of the linear stage uses two laser interferometers [16] and three capacitive sensors [14] is given in Fig. 4. The stationary part consists of a single and a dualbeam laser interferometer and three capacitive sensors perpendicular to each other. Both translational and rotational errors can be derived out of the displacement values captured by the transducers.

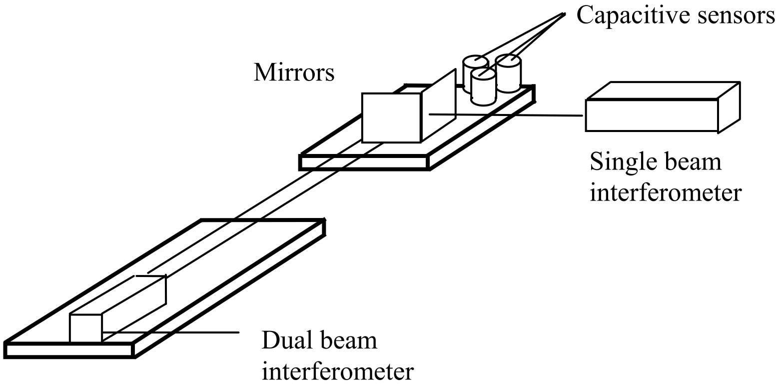

Figure 4
Set up for determining the motion error of a linear stage

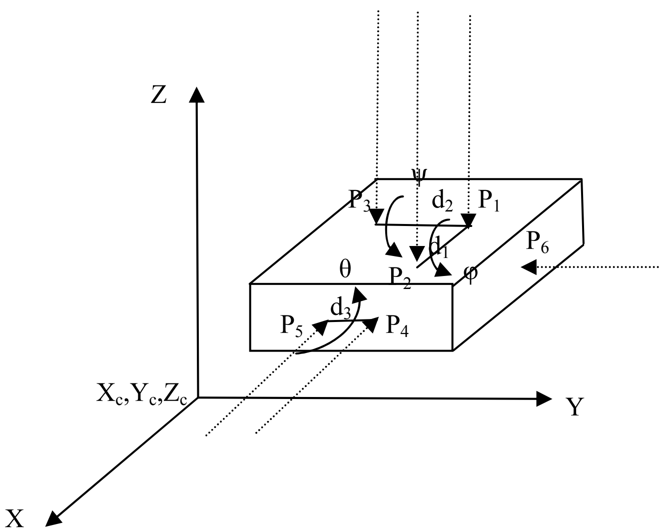

Figure 5
The measuring points on the surface of the artifact and their relations to each other

By solving the following equations the origo of the new coordinate system and its principal axis are computed. Having these values one can directly draw up the transformation matrix.

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ d_1 & 0 & d_1 tg\varphi \\ 0 & d_2 & d_2 tg\psi \end{vmatrix} = 0 \qquad \begin{vmatrix} x-x_4 & y-y_4 & z-z_4 \\ d_3 tg\theta & 0 & d_3 \\ tg\varphi & tg\psi & -1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-x_6 & y-y_6 & z-z_6 \\ tg\varphi & tg\psi & -1 \\ tg\psi & tg\theta - tg\varphi & tg\theta\, tg\psi \end{vmatrix} = 0$$

where φ, ψ and θ are angles between the priciple axis of the original coordinate system and the planes of the carridge (artifact used for the measurement).

5 Error Compensation Schemes

B spline tensor surfaces play an important role in computer aided surface design [5]. A B-spline surface can be written as:

$$S(t,u) = \sum_i \sum_j d_{ij} N_i^3(t) N_j^3(u)$$

where it was assumed that one knot sequence is along u-lin while the other one in the t direction.

This description can be extended to volumes:

$$V(t,u,v) = \sum_i \sum_j \sum_k \; d_{ij} N_i^3(t) N_j^3(u) N_k^3(v)$$

If we consider the values V as the error vector and the parameters t, u and v as the coordinites of the ideal position then by determining the weights a continouos volumetric representation of the resulting errors can be obtained. The weigth are computed using a least square procedure [9]. The subsequent figure illustrate this approach. The draw back of this approach is the same as the interpolation schemes presented in the literature, namely that they do not preserve the orthogonality of the rotation matrices and therefore they do not represent pure rotation. In the subsequent paragraphs we suggest an other solution to this problem based on the application of quaternions.

Quaternions [1] were invented by Sir William Hamilton in 1843. He realized that four number are needed to describe a rotation followed by a scaling. One number describes the size of scaling, one the number of degrees to be rotated, and the lst two numbers give the plane in which the vector should be rotated. Quaternions consists of a scalar part s∈$\mathbb{R}$ and **v** = (x,y,z)∈$\mathbb{R}^3$:

$$q \equiv [s,\mathbf{v}] \equiv [s,(x,y,z)] \equiv s + \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$$

where

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1,\ \mathbf{ij} = \mathbf{k} \text{ and } \mathbf{ji} = -\mathbf{k}$$

If q is a quaternion with q = [cosθ,sinθ**n**] and p is a quartenion p = [0,**r**] then p' = qpq^{-1} is p rotated 2θ about the axis **n**.

Given a transformation matrix M the corresponding unit quaternion is can be calculated in two steps: first we must find s which is equal to:

$$s = \pm\frac{1}{2}\sqrt{M_{11} + M_{22} + M_{33} + M_{44}}$$

Now the other values follow:

$$x = \frac{M_{32} - M_{23}}{4s}$$
$$y = \frac{M_{13} - M_{31}}{4s}$$
$$z = \frac{M_{21} - M_{12}}{4s}$$

A socalled spherical linear quaternion interpolation (Slerp) can used to compute the intermediate quaternions. The quaternions generated by Slerp are unit quaternions, which means that they represent pure rotation matrices. The formula for for Slerp is:

$$\cos(\Omega) = q_o \bullet q_1$$
$$Slerp(q_0, q_1, h) = \frac{q_0 \sin((1-h)\Omega) + q_1 \sin(h\Omega)}{\sin(\Omega)}$$

where $\bullet$ stands for the inner product defined as $q \bullet q' = ss' + xx' + yy' + zz'$

An even better (smoother) interpolation can be formulated which is the spherical cubic equivalent of a Beziér curve. This is called Squad and this defined by:

Let q_1, ..., q_n point on the unit sphere. Find the cubic spline which interpolate the points in the given sequence. This can be achived by the following formula:

Squad (q_i, q_{i+1}, s_i, s_{i+1}, h) = **Slerp**(**Slerp**(q_i, q_{i+1}, h), **Slerp**(s_i, s_{i+1}, h), 2h(1-h))

where s_i are

$$s_i = q_i \cdot \exp\left(-\frac{\log\cdot(q_i^{-1}q_{i+1}) + \log\cdot(q_i^{-1}q_{i-1})}{4}\right)$$

where logq and exp q are defined as follows:

if q = [cosθ,sinθ**v**] then logq ≡ [0,θ**v**] and if q = [0,θ**v**] then expq ≡ [cosθ,sinθ**v**]

The suggested procedure to find the intermediate rotation matrices consists of the following steps:

- Convert the matrices captured by the procedure described in paragraph 4. in quaternions
- Find the **Squad** interpolation of these points
- Convert the quaterinon splines back into matrix form

On this way a matrix function representing pure transformation of the object will be obtained. The translationel error can be handled componentwise by finding planer interpolation splines. Hereby a complete matrix function can be constructed.

Conclusions

The paper presents a new approach to the compensation of geometric errors in coordinate measuring machines. It consists of a measuring procedure which captures simultaneously the six error components of moving rigid body. The transformation matrices obtained on this way are interpolated by using quaternion representation. Hereby the orthonormality of the rotation matrices are maintained. Simulation values with randomly generated error components show that the intermediate values lead to a better accuracy inprovement.

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