Uniform observer design for linear parameter varying systems

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Abstract: This paper presents a case study how to apply the recently proposed TP model transformation technique, that has been introduced for nonlinear state-feedback control design, to nonlinear observer design. The study is conducted through an example. This example treats the question of observer design to the prototypical aeroelastic wing section with structural nonlinearity. This type of model has been traditionally used for the theoretical as well as experimental analysis of two- dimensional aeroelastic behavior. The model investigated in the paper describes the nonlinear plunge and pitch motion of a wing, and exhibits complex nonlinear behavior. In preliminary works this prototypical aeroelastic wing section was stabilized by a state-feedback controller designed via TP model transformation and linear matrix inequalities. Numerical simulations are used to provide empirical validation of the resulting observer.

1 Introduction

The main goal of the paper is to study how to apply the TP (Tensor Product) model transformation to observer design. The motivation of this goal is that the TP model transformation was proposed under the Parallel Distributed Compensation (PDC) design framework [1] for nonlinear state feedback controller design [2, 3]. The TP model transformation is capable of transforming a given time varying (parameter dependent, where the parameters may include state variables) linear state-space model into time varying convex combination of finite number of linear time invariant models. The resulting linear time invariant models can then be readily substituted into Linear Matrix Inequalities (LMI), available under the PDC design framework, to determine a time varying (parameter dependent, where the parameters may include state variables) nonlinear controller according to given control specifications. This paper studies how to apply the result of the TP model transformation to observer design under the PDC design framework similarly to the controller design.

2 Nomenclature

This section is devoted to introduce the notations being used in this paper: $\{a,b,\dots\}$: scalar values, $\{\mathbf{a},\mathbf{b},\dots\}$: vectors, $\{\mathbf{A},\mathbf{B},\dots\}$: matrices, $\{\mathcal{A},\mathcal{B},\dots\}$: tensors.

$\mathbb{R}^{I_1\times I_2\times\cdots\times I_N}$: vector space of real valued $(I_1\times I_2\times\cdots\times I_N)$-tensors. Subscript defines lower order: for example, an element of matrix $\mathbf{A}$ at row-column number i,j is symbolized as $(\mathbf{A})_{i,j}=a_{i,j}$. Systematically, the i-th column vector of $\mathbf{A}$ is denoted as $\mathbf{a}_i$, i.e. $\mathbf{A}=\begin{bmatrix}\mathbf{a}_1 & \mathbf{a}_2 & \cdots\end{bmatrix}$. $\diamond_{i,j,n},\dots$: are indices. $\diamond_{I,J,N},\dots$: index upper bound: for example: $i=1..I$, $j=1..J$, $n=1..N$ or $i_n=1..I_n$. $\mathbf{A}_{(n)}$: n-mode matrix of tensor $\mathcal{A}\in\mathbb{R}^{I_1\times I_2\times\cdots\times I_N}$. $\mathcal{A}\times_n\mathbf{U}$: n-mode matrix-tensor product. $\mathcal{A}\otimes_n\mathbf{U}_n$: multiple product as $\mathcal{A}\times_1\mathbf{U}_1\times_2\mathbf{U}_2\times_3..\times_N\mathbf{U}_N$. Detailed discussion of tensor notations and operations is given in [7].

3 Basic concepts

The detailed description of the TP model transformation and PDC design framework is beyond the scope of this paper and can be found in [1, 2, 3, 4]. In the followings a few concepts are presented being used in this paper, for more details see [1, 2, 3, 4].

3.1 Parameter-varying state-space model

Consider parameter-varying state-space model:

$$\dot{\mathbf{x}}(t)=\mathbf{A}(\mathbf{p}(t))\mathbf{x}(t)+\mathbf{B}(\mathbf{p}(t))\mathbf{u}(t) \tag{1}$$
$$\mathbf{y}(t)=\mathbf{C}(\mathbf{p}(t))\mathbf{x}(t)+\mathbf{D}(\mathbf{p}(t))\mathbf{u}(t),$$

with input $\mathbf{u}(t)$, output $\mathbf{y}(t)$ and state vector $\mathbf{x}(t)$. The system matrix

$$\mathbf{S}(\mathbf{p}(t))=\begin{pmatrix}\mathbf{A}(\mathbf{p}(t)) & \mathbf{B}(\mathbf{p}(t))\\ \mathbf{C}(\mathbf{p}(t)) & \mathbf{D}(\mathbf{p}(t))\end{pmatrix}\in\mathbb{R}^{O\times I} \tag{2}$$

is a parameter-varying object, where $\mathbf{p}(t)\in\Omega$ is time varying $N-$dimensional parameter vector, where $\Omega=[a_1,b_1]\times[a_2,b_2]\times..\times[a_N,b_N]\subset\mathbb{R}^N$ is a closed hypercube. $\mathbf{p}(t)$ can also include some (or all) elements of $\mathbf{x}(t)$.

3.2 Convex state-space TP model

Equ. (2) can be approximated for any parameter $\mathbf{p}(t)$ as a convex combination of the R number of LTI system matrices $\mathbf{S}_r$, $r=1..R$. Matrices $\mathbf{S}_r$ are also termed as vertex system matrices. Therefore, one can define weighting functions $w_r(\mathbf{p}(t))\in[0,1]\subset\mathbb{R}$ such that matrix $\mathbf{S}(\mathbf{p}(t))$ belongs to the convex hull of $\mathbf{S}_r$ as $\mathbf{S}(\mathbf{p}(t))=co\{\mathbf{S}_1,\mathbf{S}_2,..,\mathbf{S}_R\}_{\mathbf{w}(\mathbf{p}(t))}$, where vector $\mathbf{w}(\mathbf{p}(t))$ contains the weighting functions $w_r(\mathbf{p}(t))$ of the convex combination. The control design methodology, to be applied in this paper, uses univariate weighting functions. Thus, the explicit form of the convex combination in terms of tensor product becomes:

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \end{pmatrix} \approx \tag{3}$$

$$\left(\sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} .. \sum_{i_N=1}^{I_N} \prod_{n=1}^{N} w_{n,i_n}(p_n(t)) \mathbf{S}_{i_1,i_2,..,i_N} \right) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix}.$$

(3) is termed as TP model in this paper. Function $w_{n,j}(p_n(t)) \in [0,1]$ is the j-th univariate weighting function defined on the n-th dimension of Ω, and $p_n(t)$ is the n-th element of vector $\mathbf{p}(t)$. I_n (n=1,...,N) is the number of univariate weighting functions used in the n-th dimension of the parameter vector $\mathbf{p}(t)$. The multiple index $(i_1, i_2, ..., i_N)$ refers to the LTI system corresponding to the i_n-th weighting function in the n-th dimension. Hence, the number of LTI vertex systems $\mathbf{S}_{i_1,i_2,..,i_N}$ is obviously $R = \prod_n I_n$. One can rewrite (3) in the concise TP form as:

$$\begin{pmatrix} s\mathbf{x}(t) \\ \mathbf{y}(t) \end{pmatrix} \approx \mathcal{S} \underset{n=1}{\overset{N}{\otimes}} \mathbf{w}_n(p_n(t)) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix}, \tag{4}$$

that is

$$\mathbf{S}(\mathbf{p}(t)) \underset{\varepsilon}{\approx} \mathcal{S} \underset{n=1}{\overset{N}{\otimes}} \mathbf{w}_n(p_n(t)).$$

Here, ε represents the approximation error, and row vector $\mathbf{w}_n(p_n) \in \mathbb{R}^{I_n}$ contains the weighting functions $w_{n,i_n}(p_n)$, the $N+2$ -dimensional coefficient tensor $S \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N \times O \times I}$ is constructed from the LTI vertex system matrices $\mathbf{S}_{i_1,i_2,...,i_N} \in \mathbb{R}^{O \times I}$. The first N dimensions of S are assigned to the dimensions of Ω. The convex combination of the LTI vertex systems is ensured by the conditions:

Definition 1 *The TP model (4) is convex if:*

$$\forall n, i, p_n(t) : w_{n,i}(p_n(t)) \in [0,1]; \tag{5}$$

$$\forall n, p_n(t) : \sum_{i=1}^{I_n} w_{n,i}(p_n(t)) = 1. \tag{6}$$

This simply means that $\mathbf{S}(\mathbf{p}(t))$ is within the convex hull of LTI vertex systems $\mathbf{S}_{i_1,i_2,..,i_N}$ for any $\mathbf{p}(t) \in \Omega$.

4 Model of the prototypical aeroelastic wing section

In the past few years various studies of aeroelastic systems have emerged. [10] presents a detailed background and refers to a number of papers dealing with the modelling and control of aeroelastic systems. The following provides a brief summary of this background. [11] and [12] proposed non-linear feedback control methodologies for a class of non-linear structural effects of the wing section [13]. Papers [11, 14, 10] develop a controller, capable of ensuring local asymptotic stability, via partial feedback linearization. It has been shown that by applying two control surfaces global stabilization can be achieved. For instance, global feedback linearization technique were introduced for two control actuators in the work of [10]. TP model transformation based control design was introduced in [4, 5, 6]. This control design ensures asymptotic stability with one control surface and is capable of involving various control specification beyond stability.

4.1 Equations of Motion

In this paper, we consider the problem of flutter suppression for the prototypical aeroelastic wing section as shown in Figure 1. The aerofoil is constrained to have two degrees of freedom, the plunge h and pitch α. The equations of motion of the system have been derived in many references (for example, see [15], and [16]), and can be written as

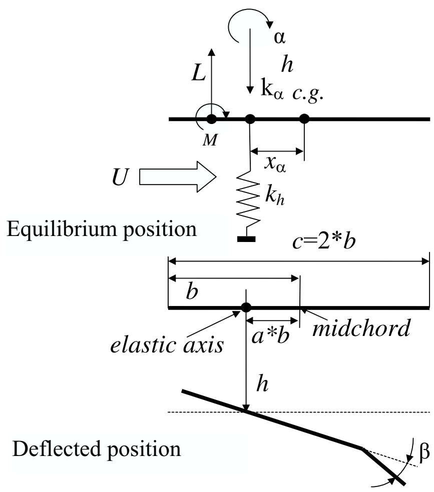

Figure 1: Aeroelastic model

$$\mathbf{x}(t) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} h \\ \alpha \\ \dot{h} \\ \dot{\alpha} \end{pmatrix} \quad \text{and} \quad \mathbf{u}(t) = \beta.$$

Then we have:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t) = \mathbf{S}(\mathbf{p}(t)) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix}, \tag{7}$$

where

$$\mathbf{A}(\mathbf{p}(t)) = \begin{pmatrix} x_3 \\ x_4 \\ -k_1x_1 - (k_2U^2 + p(x_2))x_2 - c_1x_3 - c_2x_4 \\ -k_3x_1 - (k_4U^2 + q(x_2))x_2 - c_3x_3 - c_4x_4 \end{pmatrix}$$

$$\mathbf{B}(\mathbf{p}(t)) = \begin{pmatrix} 0 \\ 0 \\ g_3U^2 \\ g_4U^2 \end{pmatrix},$$

where $\mathbf{p}(t) \in \mathbb{R}^{N=2}$ contains values x_2 and U. The parameters and variables are given in the Appendix. One should note that, the equations of motion are also dependent upon the elastic axis location a. In this paper we assume the quasi-steady aerodynamic force and moment accurate for the class of low velocities, see work [15]. The equations of motion (7) exhibit limit cycle oscillation, as well as other non-linear response regimes including chaotic response [18, 19, 21]. Papers [13, 21] have shown the relations between limit cycle oscillation, magnitudes and initial conditions or flow velocities. The system parameters to be used in this paper are given in the Appendix and are obtained from experimental models described in full detail in works [10, 21].

5 Observer design

The recently proposed very powerful numerical methods (and associated theory) for *convex optimization* involving Linear Matrix Inequalities (LMI) help us with the analysis and the design issues of dynamic systems models in acceptable computational time [22]. One direction of these analysis and design methods is based on LMI's under the PDC design framework [1]. In this paper we apply the TP model transformation in combination with the PDC based observer design technique to derive viable observer methodologies for the prototypical aeroelastic wing section defined in the previous section. The key idea of the proposed design method is that the TP model transformation is utilized to represent the model (7) in convex TP model form with specific characteristics, whereupon PDC controller design techniques can immediately be executed. The following sections introduces the observer design:

5.1 TP model form of the prototypical aeroelastic wing section

5.1.1 TP model transformation

The goal of the TP model transformation is to transform a given state-space model (1) into convex TP model [2, 3, 4], in which the LTI systems form a tight convex hull. Namely, the TP model transformation results in (4) with conditions (5) and (6), and searches the LTI systems as a points of a tight convex hull of $\mathbf{S}(\mathbf{p}(t))$.

The detailed description of the TP model transformation is discussed in [2, 3, 4]. In the followings only the main steps are briefly presented. The TP model transformation is a numerical method and has three key steps. The first step is the discreatisation of the given $\mathbf{S}(\mathbf{p}(t))$ via the sampling of $\mathbf{S}(\mathbf{p}(t))$ over a huge number of points $\mathbf{p} \in \Omega$, where Ω is the transformation space. The sampling points are defined by a dense hyper rectangular grid. In order to loose minimal information during the discretisation we apply as dense grid as possible. The second step extracts the LTI vertex systems from the sampled systems. This step is specialized to find the minimal number of LTI vertex systems, as the vertex points of the tight convex hull of the sampled systems. The third step constructs the TP model based on the LTI vertex systems obtained in the second step. It defines the continuous weighting functions to the LTI vertex systems.

5.2 Determination of the convex TP model form of the aeroelastic model

We execute the TP model transformation on the model (7). First of all, according to the three steps of the TP model transformation, let us define the transformation space Ω. We are interested in the interval $U \in [14,25](m/s)$ and we presume that, the interval $\alpha \in [-0.1,0.1](rad)$ is sufficiently large enough. Therefore, let: $\Omega : [14,25] \times [-0.1,0.1]$ in the present example (note that these intervals can arbitrarily be defined). Let the grid density be defined as $M_1 \times M_2$, $M_1 = 100$ and $M_2 = 100$. Step 2 of the TP model transformation yields 6 vertex LTI systems.

The third step results in weighting functions $w_{1,i}(U)$ and $w_{2,j}(\alpha)$.

6 Observer design to the prototypical aeroelastic wing section

6.1 Method for observer design under PDC framework

In reality not all the state variables are readily available in most cases. Unavailable state variables should be estimated in the case of state-feedback control strategy. Under these circumstances, the question arises whether it is possible to determine the state from the system response to some input over some period of time. Namely, the observer is required to satisfy:

$$\mathbf{x}(t) - \hat{\mathbf{x}}(t) \to 0 \quad \text{as} \quad t \to \infty,$$

where $\hat{\mathbf{x}}(t)$ denotes the state vector estimated by the observer. This condition guaranties that the steady-state error between $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$ converges to 0. We use the following observer structure:

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}(\mathbf{p}(t))\hat{\mathbf{x}}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t) + \mathbf{K}(\mathbf{p}(t))(\mathbf{y}(t) - \hat{\mathbf{y}}(t))$$

$$\hat{\mathbf{y}}(t) = \mathbf{C}(\mathbf{p}(t))\hat{\mathbf{x}}(t),$$

That is in TP model form:

$$\dot{\hat{\mathbf{x}}}(t) = \mathcal{A}\bigotimes_n \mathbf{w}(p_n(t))\hat{\mathbf{x}}(t) + \mathcal{B}\bigotimes_n \mathbf{w}_n(p_n(t))\mathbf{u}(t)+ \tag{8}$$

$$+\mathcal{K}\bigotimes_n \mathbf{w}(p_n(t))(\mathbf{y}(t) - \hat{\mathbf{y}}(t))$$

$$\hat{\mathbf{y}}(t) = \mathcal{C}\bigotimes_n \mathbf{w}(p_n(t))\hat{\mathbf{x}}(t).$$

At this point, we should emphasize that in our example the vector $\mathbf{p}(t)$ does not contain values form the estimated state-vector $\hat{\mathbf{x}}(t)$, since $p_1(t)$ equals U and $p_2(t)$ equals the pitch angle $(x_2(t))$. These variables are observable. We estimate only state-values $x_3(t)$ and $x_4(t)$. Consequently, the goal in the present case, is to determine gains in tensor $\mathcal{K}$ for (8). For this goal, the following LMI theorem can be find in [1]. Before dealing with this LMI theorem, we introduce a simple indexing technique, in order, to have direct link between the TP model form (4) and the typical form of LMI formulations:

Method 1 *(*Index transformation) *Let*

$$\mathbf{S}_r = \begin{pmatrix} \mathbf{A}_r & \mathbf{B}_r \\ \mathbf{C}_r & \mathbf{D}_r \end{pmatrix} = \mathbf{S}_{i_1,i_2,..,i_N},$$

where $r = ordering(i_1, i_2, .., i_N)$ *(*$r = 1..R = \prod_n I_n$*). The function "ordering" results in the linear index equivalent of an N dimensional array's index* $i_1, i_2, .., i_N$*, when the size of the array is* $I_1 \times I_2 \times .. \times I_N$*. Let the weighting functions be defined according to the sequence of r:*

$$w_r(\mathbf{p}(t)) = \prod_n w_{n,i_n}(p_n(t)).$$

Theorem 1 *(*Globally and asymptotically stable observer)

In order to ensure

$$\mathbf{x}(t) - \hat{\mathbf{x}}(t) \to 0 \quad as \quad t \to \infty,$$

in the observer strategy (8), find $\mathbf{P} > 0$ *and* $\mathbf{N}_r$ *satisfying the following LMI's.*

$$-\mathbf{A}_r^T\mathbf{P} - \mathbf{P}\mathbf{A}_r + \mathbf{C}_r^T\mathbf{N}_r^T + \mathbf{N}_r\mathbf{C}_r > 0 \tag{9}$$

for all r and

$$-\mathbf{A}_r^T\mathbf{P} - \mathbf{P}\mathbf{A}_r - \mathbf{A}_s^T\mathbf{P} - \mathbf{P}\mathbf{A}_s+ \tag{10}$$

$$+\mathbf{C}_r^T\mathbf{N}_s^T + \mathbf{N}_s\mathbf{C}_r + \mathbf{C}_s^T\mathbf{N}_r^T + \mathbf{N}_r\mathbf{C}_s > 0.$$

for $r < s \le R$*, except the pairs* (r,s) *such that* $w_r(\mathbf{p}(t))w_s(\mathbf{p}(t)) = 0, \forall \mathbf{p}(t)$.

Since the above equations are LMI's, with respect to variables $\mathbf{P}$ and $\mathbf{N}_r$, we can find a positive definite matrix $\mathbf{P}$ and matrix $\mathbf{N}_r$ or determine that no such matrices exist. This is a convex feasibility problem. Numerically, this problem can be solved very efficiently by means of the most powerful tools available in the mathematical programming literature e.g. MATLAB-LMI toolbox [22].

The observer gains can then be obtained as:

$$\mathbf{K}_r = \mathbf{P}^{-1}\mathbf{N}_r. \tag{11}$$

Finally, by the help of $r = ordering(i_1, i_2, .., i_N)$ in Method 1 one can define $\mathbf{K}_{i_1,i_2,..,i_N}$ from $\mathbf{K}_r$ obtained in (11) and store into tensor $\mathcal{K}$ of (8).

6.2 Observer design to the prototypical aeroelastic wing section

This section applies Theorem 1 to the TP model of the aeroelastic wing section. We define matrix $\mathbf{C}$ for all r from:

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t),$$

that is in present case:

$$\mathbf{C}_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The LMIs of Theorem 1, applied to the result of the TP model transformation, are feasible. Thus, equ. (11) yields 6 observer feedbacks. In conclusion the state values $x_3(t)$ and $x_4(t)$ are estimated by (10) as:

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}(\mathbf{p}(t))\hat{\mathbf{x}}(t) + \mathbf{B}(\mathbf{p}(t))u(t) + \\ \left(\sum_{i=1}^{3}\sum_{j=1}^{2} w_{1,i}(U)w_{2,j}(\alpha)\mathbf{k}_{i,j}\right)(\mathbf{y}(t) - \hat{\mathbf{y}}(t)),$$

where

$$\mathbf{y}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \quad \text{and} \quad \hat{\mathbf{y}}(t) = \begin{pmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{pmatrix} \quad \text{and} \quad \mathbf{p}(t) = \begin{pmatrix} U \\ \alpha \end{pmatrix},$$

($x_1(t) = h$, plunge, and $x_2(t) = \alpha$, pitch). In order to demonstrate the accuracy of the observer, numerical experiments are presented in the next section.

6.3 Simulation results

We simulate the observer for initials $\mathbf{x}(0) = \begin{pmatrix} 0.01 & 0.1 & 0.1 & 0.1 \end{pmatrix}^T$ and

$\hat{\mathbf{x}}(0) = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}^T$, for the open loop case. Figure 2 shows how the observer is capable of converging to the unmeasurable state values $x_3(t)$ and $x_4(t)$ (dashed line is estimated by the observer).

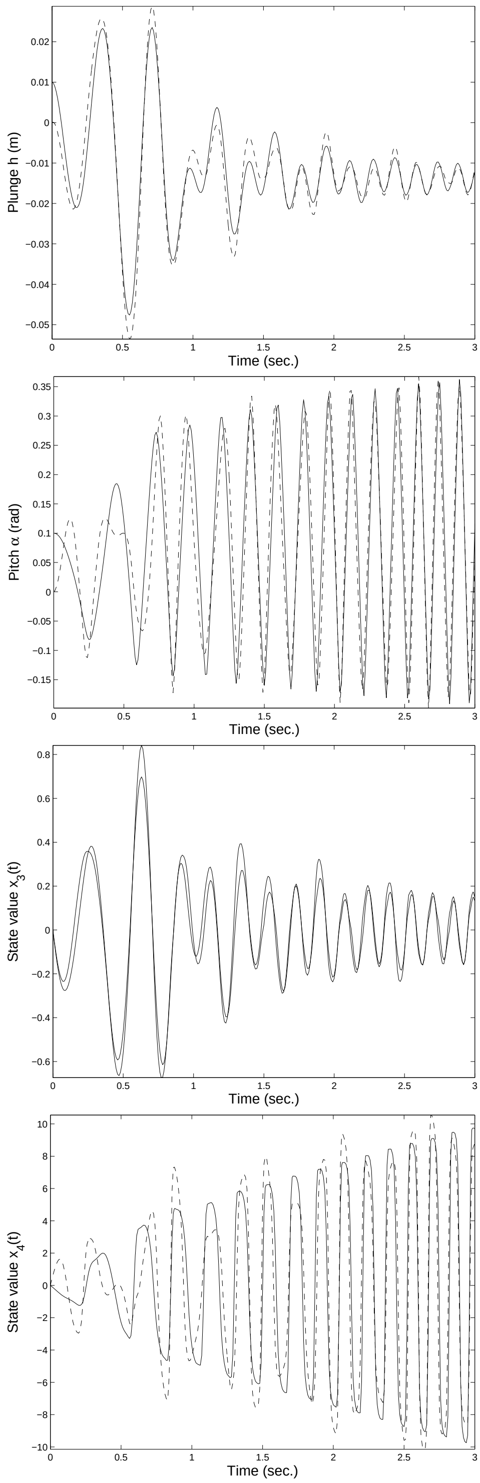

Figure 2: State values of $\mathbf{x}(t)$ (solid line) and the estimated values of $\hat{\mathbf{x}}(t)$ (dashed line) for open loop response. ($U = 20m/s$, $a = -0.4$, initials: $\mathbf{x}(0) = \begin{pmatrix} 0.01 & 0.1 & 0.1 & 0.1 \end{pmatrix}^T$, $\hat{\mathbf{x}}(0) = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}^T$)

7 Conclusion

The paper presents how to use the TP model transformation method can be used for observer design in uniform way for controller and observer design. The paper also shows how to determine observer for the prototypical aeroelastic wing section.

Appendix

x_α is the non-dimensional distance between elastic axis and the centre of mass; m is the mass of the wing; I_α is the mass moment of inertia; b is semi-chord of the wing, and c_α and c_h respectively are the pitch and plunge structural damping coefficients, and k_h is the plunge structural spring constant. ρ is the air density, U is the free stream velocity, c_{l_α} and c_{m_α} respectively, are lift and moment coefficients per angle of attack, and c_{l_β} and c_{m_β}, respectively are lift and moment coefficients per control surface deflection, and a is non-dimensional distance from the mid-chord to the elastic axis. β is the control surface deflection.

Several classes of non-linear stiffness contributions $k_\alpha(\alpha)$ have been studied in papers treating the open-loop dynamics of aeroelastic systems [17, 18, 19, 20]. In this paper we use non-linear stiffness term $k_\alpha(\alpha)$ as obtained by curve-fitting on the measured displacement-moment data for non-linear spring as [21]:

$$k_\alpha(\alpha) = 2.82(1 - 22.1\alpha + 1315.5\alpha^2 + 8580\alpha^3 + 17289.7\alpha^4).$$

System parameters

$b = 0.135m$; $span = 0.6m$; $k_h = 2844.4N/m$; $c_h = 27.43Ns/m$; $c_\alpha = 0.036Ns$; $\rho = 1.225kg/m^3$; $c_{l_\alpha} = 6.28$; $c_{l_\beta} = 3.358$; $c_{m_\alpha} = (0.5 + a)c_{l_\alpha}$; $c_{m_\beta} = -0.635$; $m = 12.387kg$; $x_\alpha = -0.3533 - a$; $I_\alpha = 0.065kgm^2$; $c_\alpha = 0.036$;

System variables

$d = m(I_\alpha - mx_\alpha^2 b^2)$; $k_1 = \frac{I_\alpha k_h}{d}$; $k_2 = \frac{I_\alpha \rho b c_{l_\alpha} + m x_\alpha b^3 \rho c_{m_\alpha}}{d}$; $k_3 = \frac{-m x_\alpha b k_h}{d}$;
$k_4 = \frac{-m x_\alpha b^2 \rho c_{l_\alpha} - m\rho b^2 c_{m_\alpha}}{d}$; $p(\alpha) = \frac{-m x_\alpha b}{d} k_\alpha(\alpha)$; $q(\alpha) = \frac{m}{d} k_\alpha(\alpha)$;
$c_1(U) = \left(I_\alpha(c_h + \rho U b c_{l_\alpha}) + m x_\alpha \rho U^3 c_{m_\alpha}\right)/d$;
$c_2(U) = \left(I_\alpha \rho U b^2 c_{l_\alpha}(\frac{1}{2} - a) - m x_\alpha b c_\alpha + m x_\alpha \rho U b^4 c_{m_\alpha}(\frac{1}{2} - a)\right)/d$;
$c_3(U) = \left(-m x_\alpha b c_h - m x_\alpha \rho U b^2 c_{l_\alpha} - m\rho U b^2 c_{m_\alpha}\right)/d$;
$c_4(U) = \left(m c_\alpha - m x_\alpha \rho U b^3 c_{l_\alpha}(\frac{1}{2} - a) - m\rho U b^3 c_{m_\alpha}(\frac{1}{2} - a)\right)/d$;
$g_3 = (-I_\alpha \rho b c_{l_\beta} - m x_\alpha b^3 \rho c_{m_\beta})/d$; $g_4 = (m x_\alpha b^2 \rho c_{l_\beta} + m\rho b^2 c_{m_\beta})/d$;

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