Development of TS Fuzzy Controllers with Dynamics for Low Order Benchmarks with Time Variable Parameters

Stefan Preitl and Radu-Emil Precup

"Politehnica" University of Timisoara, Dept. of Automation and Appl. Inform.

Bd. V. Parvan 2, RO-300223 Timisoara, Romania

Phone: +40-256-4032-29, -30, -24, -26, Fax: +40-256-403214

E-mail*: spreitl@aut.utt.ro, rprecup@aut.utt.ro*

Abstract: The paper presents development and tuning solutions for conventional and Takagi-Sugeno fuzzy controllers with dynamics of PI and PID type meant for electrical driving systems.Two control structures with homogenous and with non-homogenous information processing with respect to the inputs are presented including optimization aspects. Then Takagi-Sugeno fuzzy models dedicated to a class of plants characterized by Two Input-Single Output linear time-varying systems are presented. It is offered a stability test algorithm of the fuzzy control systems involving Takagi-Sugeno fuzzy controllers to control the accepted class of plants. The tuning methods are briefly presented in relation with a control solution for a drive system with a variable inertia strip winding system.

Keywords: Takagi-Sugeno fuzzy models, Takagi-Sugeno fuzzy controllers, stability analysis, winding system.

1 Introduction

Take the class of plants (P) having the transfer functions expressed as:

$$H_P(s) = \frac{k_P}{s(1+sT_\Sigma)} \text{ (a)}, \quad H_P(s) = \frac{k_P}{(1+sT_1)(1+sT_\Sigma)} \text{ (b)} \qquad (1.1)$$

$$H_P(s) = \frac{k_P}{s(1+sT_1)(1+sT_\Sigma)} \text{ (a)}, \quad H_P(s) = \frac{k_P}{s(1+sT_1)(1+sT_2)(1+sT_\Sigma)} \text{ (b)}. \qquad (1.2)$$

The parametres k_P (constant or variable) and $T_\Sigma < T_2 <(<<) T_1$ characterize well enough many control applications with electrical drives (as controlled plants) [1],[2].

The paper aim is to develop Takagi-Sugeno (TS) fuzzy controllers (FCs) based on classical development methods, meant for controlling electrical drives with linear time-varying (LTV) parameters (benchmarks, (1) and (2)). LTV systems may result of linearized nonlinear systems in the vicinity of a set of operating points or of a trajectory.

These features determine the wide application area of TS fuzzy models in spite of their drawbacks such as: - the behavior of the global TS fuzzy model can significantly divert from the expected behavior obtained by the merge of the local models; - the stability analysis and testing of fuzzy control systems based on TS fuzzy models is relatively difficult because of the complex aggregation of the local models in the inference engine.

Firstly, the paper presents shortly the use of classical development procedure for PI(D) controllers (Section 2). Then, a class of TS models for Two Input-Single Output (TISO) LTV plants is presented (Section 3). In Section 4 there are defined the TS fuzzy controllers meant for controlling the TS fuzzy models. Based on these is presented a stability test algorithm (based on Lyapunov's stability theory) for a class of fuzzy systems with TS fuzzy controllers controlling the TISO LTV plants (Section 5). Results concerning the development of conventional and fuzzy control solutions for a drive system with two output coupled motors applicable to the rolls of a hot rolling mill and to a variable inertia strip winding system are presented in Section 6. Section 7 is focused on the concluding part of the paper.

2 Development of Continuously and Quasi-Continuously Operating PI(D) Control Algorithms

Many control applications prefer structures with typical control algorithms with homogenous or non-homogenous information processing on the two input channels [1]. Such structures have the general form given in Fig.2.1-a -b and -c presents some particular control laws regarding the inputs. There can be established relations between such controllers and the 2-DOF controllers [3]. The blocks (1) ... (5) can be described by its specific transfer functions (t.f.s).

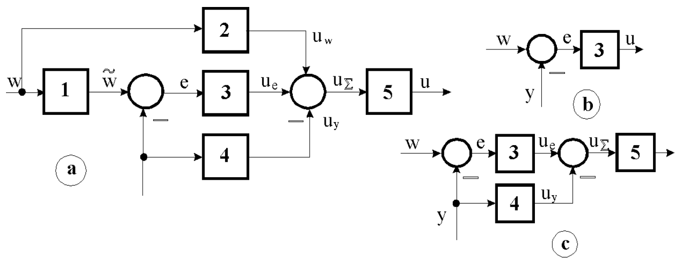

Fig.2.1. Typical controller structures and particularizations regarding the modules.

In the presence of an integral (I) component and a limitation block in the controller structure, the use of the AWR measure (Anti-Windup-Reset) will be recommended. The transfer functions of the continuous PI(D) controllers are written related to the design procedure and the implementation (discretization) procedure; some well-known forms are:

PI: $\{k_r, T_r\}$ $$H_R(s) = \frac{k_r}{s}(1 + sT_r) \quad (2.1)$$

PID: $\{k_r, T_r, T_r'\}$ $$H_R(s) = \frac{k_r}{s}(1 + sT_r)(1 + sT_r') \quad (2.2)$$

The implementation of a quasi-continuously (QC) operating PID digital control algorithm can be based on the informational diagram presented in Fig.2.2; the appearance of a supplementary state variables x_k, is associated to the I component and the adding of the AWR measure. The *parameter values* $\{K_{pid}, K_i, K_d, K_{arw}\}$ depend on the continuous parameters $\{k_r, T_r, T_r'\}$ and on the sampling time value, T_e [4].

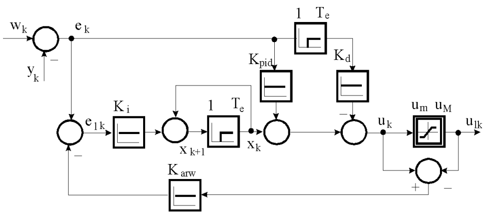

Fig.2.2. A quasi-continuously operating PID digital control algorithm implementation.

The implementation of non-homogenous information processing (Fig.2.1-c) has two requirements [5]: - an I or PI behavior with respect to the reference channel; - a PI or PID behavior with respect to the feedback channel. The non-homogenous information processing structure respects the following informations (Table 2.1).

Table 2.1. Transfer functions of blocks in Fig.2.1-c

Case	Channel	Block 3	Block 4	Block 5	Type
(1)	*w*	I: $(1/sT_i)$	----	P: (k_R)	I: $(1/sT_i)$
	y	I: $(1/sT_i)$	P: (1)	P: (k_R)	PI: $(1+1/sT_i)$
(2)	*w*	PI: $(1+1/sT_i)$	----	P: (k_R)	PI: $(1+1/sT_i)$
	y	PI: $(1+1/sT_i)$	D: (sT_d)	P: (k_R)	PID

The parameters can be calculated using the relations synthetised in Table 2.2. The parameter β belongs usually to the domain $4 \le \beta \le 16$.

3 A Class of Takagi-Sugeno Fuzzy Models

The following Takagi-Sugeno fuzzy model to represent a TISO LTV system will be used that models the controlled plant [7]:

$$\begin{aligned} R^l: &\text{IF } z_1(t) \text{ is } F_1^l \text{ AND } z_2(t) \text{ is } F_2^l \text{ AND } \dots \text{ AND } z_n(t) \text{ is } F_n^l \\ &\text{THEN } y(s) = H_{P,l}^u(s)u(s) + H_{P,l}^v(s)v(s),\ l = 1 \dots m, \end{aligned} \quad (3.1)$$

Table 2.2. Tuning relations after [2], [5],[6]

$H_P(s)$	Contr. type	Tuning relations
0	1	2
$\dfrac{k_p}{s(1+sT_\Sigma)}$	PI $\{k_r, T_r\}$	$k_r = \dfrac{1}{\beta^{3/2} k_p T_\Sigma^2}$ $T_r = \beta T_\Sigma$
$\dfrac{k_p}{s(1+sT_1)(1+sT_\Sigma)}$	PID $\{k_r, T_r, T_r'\}$	$k_r = \dfrac{1}{\beta^{3/2} k_p T_\Sigma^2}$ $T_r = \beta T_\Sigma$, $T_r' = T_1$
$H_P(s) = \dfrac{k_P}{s(1+sT_1)(1+sT_\Sigma)}$ $m = T_\Sigma / T_1$	PI $\{k_r, T_r\}$	$k_r = \dfrac{(1+m)^2}{\beta^{3/2} \cdot k_P \cdot T_\Sigma' \cdot m}$, $T_r = \dfrac{\beta\, T_\Sigma'\, \Delta_m(m)}{(1+m)^2}$
$H_P(s) = \dfrac{k_P}{s(1+sT_1)(1+sT_2)(1+sT_\Sigma)}$ $m = T_\Sigma / T_1$	PID $\{k_r, T_r, T_r'\}$	Identical with the case of PI controller tuning rel. and $T_r' = T_1$

where: $u(s)$, $v(s)$, $y(s)$ - the Laplace transform of the plant input (the control signal) $u(t)$, of the disturbance input $v(t)$ and of the controlled output $y(t)$; m – the number of inference rules; R^l – the lth inference rule, $l = 1 \dots m$; n – the number of measurable plant (system) variables pointing out the time-variation of the plant; $z_i(t)$ – the measurable plant variables, $i = 1 \dots n$, and:

$$\mathbf{z}(t) = [z_1(t)\ z_2(t)\ \dots\ z_n(t)]^T ; \tag{3.2}$$

F^l – the linguistic terms associated to the measurable variable $z^i(t)$ and to the rule R^l; $H^u_{P,l}(s)$ and $H^v_{P,l}(s)$ – the local t.f.s of the plant.

The TS fuzzy model (3.1) includs both the inference rules as part of the rule base and the local analytic models of the TISO LTV system. The controlled output is inferred by taking the weighted average of all local models appearing in (3.1), which characterizes the properties of the controlled plant in a local region of the input space; so it is referred to as fuzzy dynamic local model [7],[8]. The following notation is introduced:

$$\mu_l(t) = \mu_l(\mathbf{z}(t)),\ l = 1 \dots m , \tag{3.3}$$

for the membership degrees of the normalized membership functions μ_l of the inferred fuzzy set F^l, where:

$$F^l = \bigcap_{i=1}^{n} F_i^l, \ l = 1 \ldots m, \quad \text{(a)} \quad \text{and} \quad \sum_{l=1}^{m} \mu_l(t) = 1. \quad \text{(b)} \quad (3.4)$$

By using the product inference method in (3.4) (b) and the weighted average method for defuzzification, the TS fuzzy model (3.1) can be expressed in terms of the following fuzzy dynamic global model that can be considered as TS fuzzy model of the plant:

$$y(s) = H_P^u(s)u(s) + H_P^v(s)v(s),$$
$$H_P^u(s) = \sum_{l=1}^{m} \mu_l(t) H_{P,l}^u(s), \ H_P^v(s) = \sum_{l=1}^{m} \mu_l(t) H_{P,l}^v(s). \quad (3.5)$$

The model (3.5) is LTV system because the inferred transfer functions, $H_P^u(s)$ and $H_P^v(s)$, have time-varying coefficients.

4 Takagi-Sugeno Fuzzy Controllers. Closed-Loop System Models

The TS fuzzy models (3.1) or (3.5) could be very useful in comparison with other conventional techniques in nonlinear control. This is the case of piecewise linearization [8], where the plant is linearized around a nominal operating point, and there are applied linear control techniques to the controller development. This approach divides the input space into crisp subspaces, and the result is in a non-smooth connection of the linear subsystems to build the closed-loop system model. These models are based on the division of the input space into fuzzy subspaces and use linear local models in each subspace. Furthermore, the fuzzy sets F_i^l and the inference method permit the smooth connection of the local models to build the fuzzy dynamic global model of the closed-loop system.

To control the TISO LTV plant (3.5) there is proposed a TS fuzzy controller with the following model:

$$R^l: \text{IF } z_1(t) \text{ is } F_1^l \text{ AND } z_2(t) \text{ is } F_2^l \text{ AND } \ldots \text{ AND } z_n(t) \text{ is } F_n^l, \quad (4.1)$$
$$\text{THEN } u(s) = H_{C,l}(s)e(s), \ l = 1 \ldots m,$$

where $e(s)$ is the Laplace transform of the control error $e(t) = r(t) - y(t)$; $r(t)$ is the reference input; $H_{C,l}(s)$ – the t.f. of the local controllers, $l = 1 \ldots m$.

The local controllers in (4.1) are developed for the local analytic models in (3.1) by parallel distributed compensation [9]. By the feedback connection of the plant (3.1) and of the fuzzy controller (4.1) in terms of the conventional control structure presented in Fig.4.1, the closed-loop system can be described by the following fuzzy dynamic local model:

$$R^l: \text{IF } z_1(t) \text{ is } F_1^l \text{ AND } z_2(t) \text{ is } F_2^l \text{ AND } \ldots \text{ AND } z_n(t) \text{ is } F_n^l \\ \text{THEN } y(s) = H_{r,l}(s)r(s) + H_{v,l}(s)v(s),\ l = 1 \ldots m, \quad (4.2)$$

where $H_{r,l}(s)$ and $H_{v,l}(s)$ - the local t.f.s of the closed-loop system, $l = 1 \ldots m$.

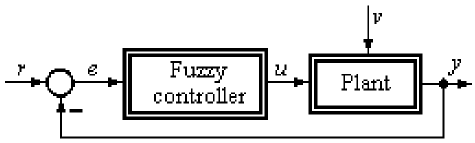

Fig.4.1. Control system structure.

In the conditions (3.3) … (3.5), by accepting the same inference method and defuzzification method as in the previous Section, the fuzzy dynamic global model of the closed-loop system can be expressed in terms of (4.3):

$$y(s) = H_r(s)r(s) + H_v(s)v(s), \\ H_r(s) = \sum_{l=1}^{m} \mu_l(t) H_{r,l}(s),\ H_v(s) = \sum_{l=1}^{m} \mu_l(t) H_{v,l}(s), \quad (4.3)$$

where the inferred t.f.s $H_r(s)$ and $H_v(s)$ have time-varying coefficients. It is justified to consider the TS fuzzy model (4.3) as TISO LTV system; for its analysis there can be applied methods specific to LTV systems [7] … [9] which require numerical techniques for the calculation of $H_r(s)$ and $H_v(s)$.

For the development of the fuzzy controllers it is necessary to perform the stability analysis and testing; a stability analysis test algorithm for the closed-loop system (4.3) are presented in the next Section.

5 Stability Test Algorithm

To perform the stability analysis of the fuzzy control systems two approaches can be employed: the first one, based on the use of the fuzzy dynamic global model (4.3) and the second one can be developed by starting with the definition of a piecewise smooth quadratic Lyapunov function [10], based on the fuzzy dynamic local model (4.2).

In the case of the system (4.2) there can be used several approaches based on either transferring the ideas from hybrid systems [8] or by using, since this system can be considered as a variable structure one with possible discontinuous right-hand side, stability analysis methods dedicated to variable structure systems [11].

For the stability analysis and testing of the fuzzy control system modeled by the fuzzy dynamic global model (4.3) it will be presented as follows the first approach, based on the Lyapunov stability theory in terms of the definition of a piecewise smooth quadratic Lyapunov function V:

$$V=\sum_{l=1}^{m} q_l V_l,\ \mathrm{V}_l=\mathbf{x}^T\mathbf{P}_l\mathbf{x}, \tag{5.1}$$

where $\mathbf{x}$ – the state vector, dim $\mathbf{x}$ = (1, n_S), $\mathbf{P}_l$ – positive definite symmetric matrices, dim $\mathbf{P}_l$ = (n_S, n_S), q_l – weighting coefficients ensuring the smoothness of the function V, l = 1 … m, n_S – system order. The matrices $\mathbf{P}_l$ are obtained by ensuring the negative definiteness of the derivative of the Lyapunov function. This can be ensured by solving the algebraic Riccati equations (5.2):

$$\mathbf{A}_l^T\mathbf{P}_l+\mathbf{P}_l\mathbf{A}_l=-\mathbf{Q}_l,\ l=1\ldots m, \tag{5.2}$$

with $\mathbf{Q}_l$ – positive definite symmetric matrices, dim $\mathbf{Q}_l$ = (n_S, n_S), and $\mathbf{A}_l$ – the system matrices in the systemic realizations corresponding to the closed-loop transfer functions $H_{r,l}(s)$ and $H_{v,l}(s)$, dim $\mathbf{A}_l$ = (n_S, n_S).

The stability analysis test algorithm consists in four steps, detailed in [atqr]. The solving of the algebraic Riccati equations (5.2) and the required analysis requires the largest computational effort.

6 Application: Winding System Control Solution

The winding process has variable inertia (Variable Inertia Drive System, VIDS) and the reference input must be correlated with the modification of work roll radius, Fig.6.1-a and –b [11]. In this context two basic aspects occur at the development of the control structure: the modification of the reference input (ω), and tuning the controller parameters.

For the first one, the condition (6.1) must be fulfilled by the control solution:

$$v(t) = \text{const} \ \rightarrow\ \omega_o(t) = k/R(t), \tag{6.1}$$

where by the measurement of $R(t)$ there can be ensured the continuous modification of the reference input $\omega_o(t)$.

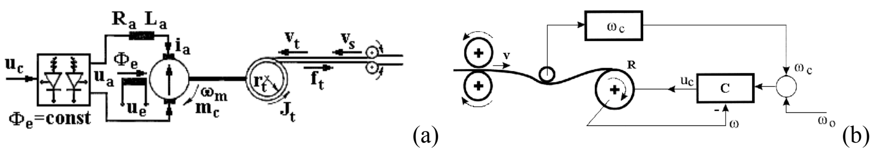

Fig.6.1. Functional diagram of VIDS and reference input correction system.

The problem of controlling the speed of the winding system can be solved in various ways: by the use of a cascade control structure with two, current and speed, controllers, or by the use of a state feedback control structure. For both versions, the variance of the moment of inertia, according to (6.2):

$$J(t) = (1/2)\,\rho\ \pi\ l\,R^4(t), \tag{6.2}$$

requires much attention in the controller design. In this paper will be presented a solutions based on control loops with linear PI and PI-fuzzy controllers with parameter adaptation.

The state-space mathematical model of VIDS has the state variables $\{x_1=i_a, x_2=\omega_m, x_3=f_t\}$, and a corresponding informational block diagram given in Fig.6.2 [11].

$$x_1'(t)=-(R_a/L_a)x_1(t)-(k_e/L_a)x_2(t)+(k_{ch}/L_a)u_c(t) ,$$
$$x_2'(t)=(k_m/J_e(t))x_1(t)-(1/J_e(t))m_f(x_2(t))-(r_t(t)/J_e(t))x_3(t))-(1/J_e(t))(J_e'(t))r_t(t),$$
$$x_3'(t)=c_b r_t(t)x_2(t)-c_b v_s(t) . \quad (6.3)$$

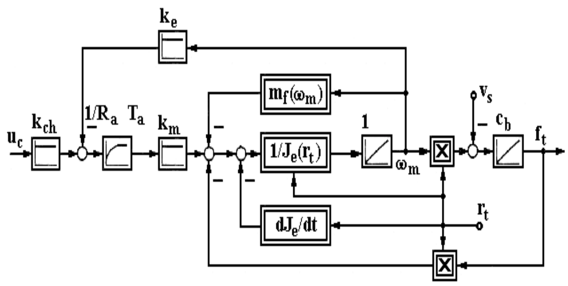

Fig.6.2. Informational block diagram of VIDS.

Linearizing the models in some representive functional points, mathematical models (benchmark t.f.s) in form of (1.1) and (1.2) will be obtained.

Concerning the local linearized plant models (1.1), (1.2) the speed controller design is based on the tuning methods described in chapter 2, applied in its various, dedicated versions [2],[4],[5]. An atractive tuning version, regarding TS fuzzy models, TS fuzzy controllers and TS fuzzy closed-loop system models (Sections 3 and 4), by accepting that the controller parameters to ensure a maximum phase reserve for each local linearised plant model and the corresponding TISO LTV systems are handled as in Sections 3-5. It can be considered equivalent with a re-tuning of the controller parameters as function of radius modification. This version permits the obtaining of better control system performance.

6.1 Cascade Control Structure

The control structure contains two loops, the inner regarded to the curent and the external regardet to the speed.

6.2 Quasi-PI Fuzzy Controllers

For the speed control loop, there are of interest the the standard quasi-PI fuzzy controllers (PI-FC) (but not only) [6]. The standard PI-FC is based on one of the structures in Fig.6.3, having as characteristic feature: for the standard PI-FC with integration on controller output (PI-FC-OI, Fig.6.3-a) the dynamics is introduced by differentiating the control error (e_k) and integrating the increment of control signal

(Δu_k), and for the standard PI-FC with integration on controller input (PI-FC-II, Fig.6.3-b) the dynamics is introduced by integrating e_k.

The membership functions for both fuzzy controllers are of the type presented in Fig.6.4 and the decision table is shown in Table 6.1.

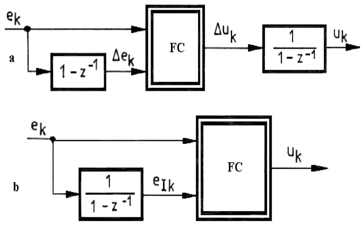

Fig.6.3. Structures of standard PI-FCs.

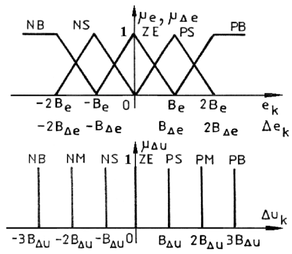

Fig.6.4. Membership functions of PI-FC-OI.

Table 6.1 Decision table of PI-FC-OI

$\Delta e_k \backslash e_k$	NB	NS	ZE	PS	PB
PB	ZE	PS	PM	PB	PB
PS	NS	ZE	PS	PM	PB
ZE	NM	NS	ZE	PS	PM
NS	NB	NM	NS	ZE	PS
NB	NB	NB	NM	NS	ZE

The parameters of these PI-FCs are $\{B_e, B_{\Delta e}, B_{\Delta u}\}$ for the standard PI-FC-OI, and $\{B_e, B_{eI}, B_u\}$ for the standard PI-FC-II. The main tuning aspects regarded to the PI(D) FCs were presented in our previous papers. A predictive version of the fuzzy controllers can be developed based on the incremental version of the digital PID controller.

Conclusions

The paper presents continuous-time development solutions for electrical drives with variable inertia. The tuning relations are deduced for classical but generally accepted benchmark type plant models.

The presented TS fuzzy models dedicated to TISO LTV systems are suitable for control structures where the plant mathematical model linearization offers local linear models.

A stability test algorithm for the fuzzy control systems modeled by TS fuzzy models based on Lyapunov stability theory. The main limitation of the stability analysis algorithm concerns its computational complexity.

The models and the stability analysis algorithm can be used in the development of TS fuzzy controllers based on the parallel distributed compensation with several applications. One real-world application can be in the area of electrical drives with variable inertia [12], [13], where the development of the local controllers can be performed in terms of the ESO method [14]. This real-world application is necessary to validate the proposed stability analysis test algorithm.

The presented application is regarded to a VIDS where the reference input must be correlated with the modification of working roll radius.

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