

Interpolation-based Fuzzy Reasoning as an Application Oriented Approach

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Abstract: Some difficulties emerging during the construction of fuzzy rule bases are inherited from the type of the applied fuzzy reasoning. The fuzzy rule base requested for many classical reasoning methods needed to be complete. In case of fetching fuzzy rules directly from expert knowledge, the way of building a complete rule base is not always straightforward. One simple solution for overcoming the necessity of the complete rule base is the application of interpolation-based fuzzy reasoning methods, since interpolation-based fuzzy reasoning methods can serve usable (interpolated) conclusion even if none of the existing rules is hit by the observation. These methods can save the expert from dealing with derivable rules and help to concentrate on cardinal actions only. For demonstrating the benefits of the interpolation-based fuzzy reasoning methods in construction of fuzzy rule bases a simple example will be introduced briefly in this paper too.

Keywords: Interpolation-based Fuzzy reasoning, rule base construction

1 Introduction

Since the classical fuzzy reasoning methods (e.g. compositional rule of inference) are demanding complete rule bases, the classical rule base construction claims a special care of filling all the possible rules. In case if there are some rules missing, there are observations may exist which hit no rule in the rule base and therefore no conclusion is obtained. Having no conclusion in a fuzzy control structure is hard to explain. E.g. one solution could be to keep the last real conclusion instead of the missing one, but applying historical data automatically to fill undeliberately missing rules could cause unpredictable side effects. Another solution for the same problem is the application of the interpolation-based fuzzy reasoning methods, where the derivable rules are deliberately missing. Since the rule base of a fuzzy interpolation-based controller, is not necessarily complete, it could contain the

most significant fuzzy rules only without risking the chance of having no conclusion for some of the observations. In other words, during the construction of the fuzzy rule base, it is enough to concentrate on the cardinal actions; the “filling” rules (rules could be deduced from the others) can be deliberately omitted.

In the followings, first an approximate fuzzy reasoning method based on interpolation in the vague environment of the fuzzy rule base [4], [5], [6] will be introduced. The main benefit of the proposed method is its simplicity, as it could be implemented to be simple and quick enough to be applied in practical direct fuzzy logic control too. Then its adaptation to fuzzy control structures through a simple rule base construction example will be discussed briefly.

2 Interpolation-based Fuzzy Reasoning

One way of interpolative fuzzy reasoning is based on the concept of vague environment [2]. Applying the idea of the vague environment the linguistic terms of the fuzzy partitions can be described by scaling functions [2] and the fuzzy reasoning itself can be replaced by classical interpolation. The concept of vague environment is based on the similarity or indistinguishability of the elements. Two values in the vague environment are ε -distinguishable if their distance is greater than ε . The distances in vague environment are weighted distances. The weighting factor or function is called *scaling function (factor)* [2]. Two values in the vague environment X are ε -distinguishable if

$$\varepsilon > \delta_s(x_1, x_2) = \left| \int_{x_2}^{x_1} s(x) dx \right|, \quad (1)$$

where $\delta_s(x_1, x_2)$ is the vague distance of the values x_1, x_2 and $s(x)$ is the scaling function on X . For finding connections between fuzzy sets and a vague environment the membership function $\mu_A(x)$ can be introduced as a level of similarity \mathbf{a} to x , as the degree to which x is indistinguishable to \mathbf{a} [2]. The α -cuts of the fuzzy set $\mu_A(x)$ are the sets which contain the elements those are $(1-\alpha)$ -indistinguishable from \mathbf{a} (see fig.1.):

$$\delta_s(\mathbf{a}, \mathbf{b}) \leq 1 - \alpha, \quad \mu_A(x) = 1 - \min\{\delta_s(\mathbf{a}, \mathbf{b}), 1\} = 1 - \min\left\{\left|\int_{\mathbf{a}}^{\mathbf{b}} s(x) dx\right|, 1\right\}. \quad (2)$$

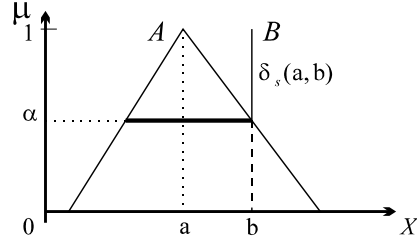


Fig. 1. The α -cuts of $\mu_A(x)$ contains the elements that are $(1-\alpha)$ -indistinguishable from a

This case (Fig.1.) the vague distance of points a and b ($\delta_s(a, b)$) is the *Disconsistency Measure* (S_D) of the fuzzy sets A and B (where B is a singleton):

$$S_D = 1 - \sup_{x \in X} \mu_{A \cap B}(x) = \delta_s(a, b) \text{ if } \delta_s(a, b) \in [0, 1], \quad (3)$$

where $A \cap B$ is the min t-norm, $\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)] \forall x \in X$.

From the viewpoint of fuzzy reasoning and fuzzy rule bases, where an observation fuzzy set is needed to be compared to rule antecedents built up member fuzzy sets (linguistic terms) of the antecedent fuzzy partitions (2) and (3) means that the disconsistency measures between member fuzzy sets of a fuzzy partition and a singleton, can be calculated as vague distances of points in the vague environment of the fuzzy partition. The main difference between the disconsistency measure and the vague distance is, that the vague distance is a value in the range of $[0, \infty]$, while the disconsistency measure is limited to $[0, 1]$.

Therefore if it is possible to describe all the fuzzy partitions of the primary fuzzy sets (the antecedent and consequent universes) of the fuzzy rule base by vague environments, and the observation is a singleton, the “extended” disconsistency measures of the antecedent primary fuzzy sets of the rule base, and the “extended” disconsistency measures of the consequent primary fuzzy sets and the consequence can be calculated as vague distances of points in the antecedent and consequent vague environments.

The vague environment is described by its scaling function. For generating a vague environment of a fuzzy partition we have to find an appropriate scaling function, which describes the shapes of all the terms in the fuzzy partition. A fuzzy partition can be characterised by a single vague environment if and only if the membership functions of the terms fulfil the following requirement [2]:

$$s(x) = |\mu'(x)| = \left| \frac{d\mu}{dx} \right| \text{ exists iff } \min\{\mu_i(x), \mu_j(x)\} > 0 \Rightarrow |\mu'_i(x)| = |\mu'_j(x)|, \quad (4)$$

$\forall i, j \in I$, where $s(x)$ is the vague environment.

Generally the above condition is not fulfilling, so the question is how to describe all fuzzy sets of the fuzzy partition with one “universal” scaling function. For this task the concept of *approximate scaling function*, as an approximation of the scaling functions describes the terms of the fuzzy partition separately [4], [5], [6] is proposed. If the vague environment of a fuzzy partition (the scaling function or the approximate scaling function) exists, the member sets of the fuzzy partition can be characterised by points in the vague environment. (These points are characterising the cores of the fuzzy terms, while the membership functions are described by the scaling function itself.) If all the vague environments of the antecedent and consequent universes of the fuzzy rule base are exist, all the primary fuzzy sets (linguistic terms) used in the fuzzy rule base can be characterised by points in their vague environment. Therefore the fuzzy rules (build on the primary fuzzy sets) can be characterised by points in the vague environment of the fuzzy rule base too. This case the approximate fuzzy reasoning can be handled as a classical interpolation task. Applying the concept of vague environment (the distances of points are weighted distances), any interpolation, extrapolation or regression methods can be adapted very simply for approximate fuzzy reasoning [4], [5], [6].

Because of its simple multidimensional applicability, for interpolation-based fuzzy reasoning in this paper the adaptation of the *Shepard operator* based interpolation (first introduced in [1]) is suggested. Beside the existing deep application oriented investigation of the Shepard operator e.g. [3], it is also successfully applied in the *Kóczy-Hirota fuzzy interpolation* [12]. (The stability and the approximation rate of the Shepard operator based Kóczy-Hirota fuzzy interpolation is deeply studied in [7] and [8].) The Shepard interpolation method for arbitrarily placed bivariate data was introduced as follows [1]:

$$S_0(f, x, y) = \begin{cases} f_k & \text{if } (x, y) = (x_k, y_k) \text{ for some } k, \\ \left(\frac{\sum_{k=0}^n f(x_k, y_k) / d_k^\lambda}{\sum_{k=0}^n 1 / d_k^\lambda} \right) & \text{otherwise,} \end{cases} \quad (5)$$

where measurement points x_k, y_k ($k \in [0, n]$) are irregularly spaced on the domain of $f \in \mathfrak{R}^2 \rightarrow \mathfrak{R}$, $\lambda > 0$, and $d_k = [(x - x_k)^2 + (y - y_k)^2]^{1/2}$. This function can be typically used when a surface model is required to interpolate scattered spatial measurements.

The adaptation of the Shepard interpolation method for interpolation-based fuzzy reasoning in the vague environment of the fuzzy rule base is straightforward by substituting the Euclidian distances with vague distances:

$$\delta_{s,k} = \delta_s(\mathbf{a}_k, \mathbf{x}) = \left[\sum_{i=1}^m \left(\int_{a_{k,i}}^{x_i} S_{X_i}(x_i) dx_i \right)^2 \right]^{1/2}, \quad (6)$$

where s_{X_i} is the i^{th} scaling function of the m dimensional antecedent universe, \mathbf{x} is the m dimensional crisp observation and \mathbf{a}_k are the cores of the m dimensional fuzzy rule antecedents A_k .

Thus in case of singleton rule consequents fuzzy rules R_k

$$\text{If } x_1 = A_{k,1} \text{ And } x_2 = A_{k,2} \text{ And } \dots \text{ And } x_m = A_{k,m} \text{ Then } y = c_k \quad (7)$$

by substituting (6) to (5) the conclusion of the interpolative fuzzy reasoning can be obtained as:

$$y(\mathbf{x}) = \begin{cases} c_k & \text{if } \mathbf{x} = \mathbf{a}_k \text{ for some } k, \\ \left(\frac{\sum_{k=1}^r c_k / \delta_{s,k}^\lambda}{\sum_{k=1}^r 1 / \delta_{s,k}^\lambda} \right) & \text{otherwise.} \end{cases} \quad (8)$$

The interpolative fuzzy reasoning (8) can simply extend to be able to handle fuzzy conclusions by introducing the vague environment (scaling function) of the consequence universe. This case the fuzzy rules R_k has the following form:

$$\text{If } x_1 = A_{k,1} \text{ And } x_2 = A_{k,2} \text{ And } \dots \text{ And } x_m = A_{k,m} \text{ Then } y = B_k \quad (9)$$

By introducing vague distances on the consequence universe:

$$\delta_s(\mathbf{b}_k, y) = \left[\left(\int_{\mathbf{b}_k}^y s_Y(y) dy \right)^2 \right]^{1/2}, \quad (10)$$

where s_Y is the i^{th} scaling function of the one dimensional consequent universe, \mathbf{b}_k are the cores of the one dimensional fuzzy rule consequents B_k .

Introducing the first element of the one dimensional consequence universe \mathbf{b}_0 the ($Y: \mathbf{b}_0 \leq y \quad \forall y \in Y$), based on (8) and (10) the requested one dimensional conclusion $y(\mathbf{x})$ can be obtained from the following formula:

$$\delta_s(y(\mathbf{x}), \mathbf{b}_0) = \begin{cases} \delta_s(\mathbf{b}_k, \mathbf{b}_0) & \text{if } \mathbf{x} = \mathbf{a}_k \text{ for some } k, \\ \left(\frac{\sum_{k=1}^r \delta_s(\mathbf{b}_k, \mathbf{b}_0) / \delta_{s,k}^\lambda}{\sum_{k=1}^r 1 / \delta_{s,k}^\lambda} \right) & \text{otherwise.} \end{cases} \quad (11)$$

A simple one-dimensional example for the approximate scaling function and the Shepard operator based interpolation (11) is introduced on Fig. 2 and on Fig. 3.

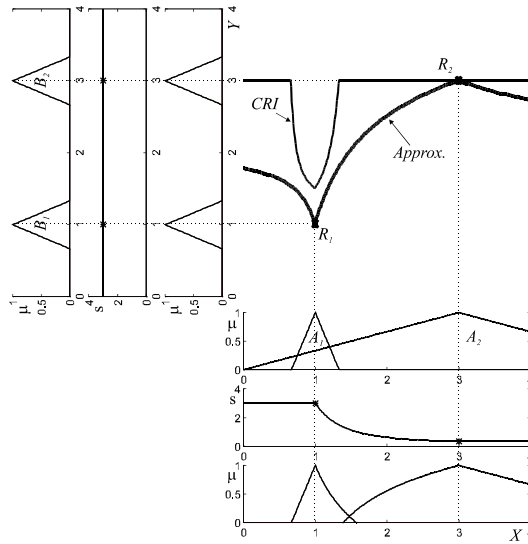


Fig. 2. Interpolation of two fuzzy rules ($R_i: A_i \rightarrow B_i$) (see fig. 3. for notation)

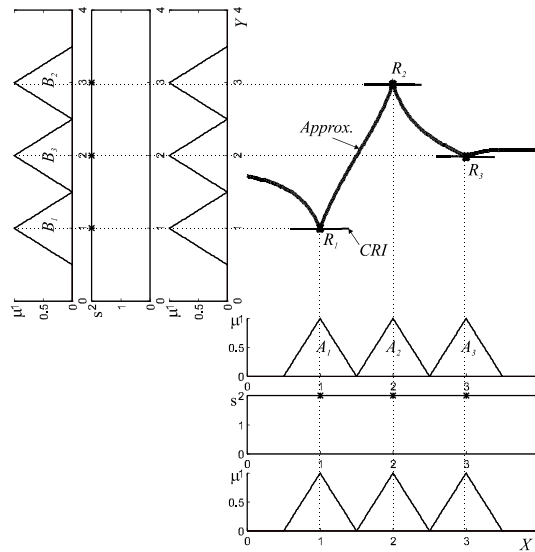


Fig. 3. Interpolation of three fuzzy rules ($R_i: A_i \rightarrow B_i$) in the approximated vague environment of the fuzzy rule base, using the Shepard operator based interpolation ($p=1$) (*Approx.*), and the min-max. CRI with the centre of gravity defuzzification (CRI), where μ is the membership grade, and s is the scaling function

For comparing the crisp conclusions of the interpolation-based fuzzy reasoning and the classical methods, the conclusions generated by the max-min

compositional rule of inference (CRI) and the centre of gravity defuzzification for the same rule base is also demonstrated on the example figures (Fig. 2, Fig. 3). More detailed description of the proposed approximate fuzzy reasoning method can be found in [4], [5], [6].

3 Application Example

The main benefit of the interpolation-based fuzzy reasoning method, introduced in the previous chapter, is its simplicity. Applying look-up tables for pre-calculating the vague distances, it could be implemented to be simple and quick enough to fit the speed requirements of practical real-time direct fuzzy logic control systems, e.g. the requirements of fuzzy behaviour-based control too. The calculation efforts of many other interpolation-based fuzzy reasoning methods “wasted” for determining the exact membership shape of the interpolated fuzzy conclusion prohibits their practical application in real-time direct fuzzy logic control. The lack of the fuzziness in the conclusion is a disadvantage of the proposed method, but it has no influence in common applications where the next step after the fuzzy reasoning is the defuzzification.

For demonstrating the simplicity of defining rule base for interpolation-based fuzzy reasoning, as an example, the construction of the state-transition rule base of a user adaptive information retrieval system will be introduced briefly in the followings.

In this user adaptive information retrieval system example (introduced in [10] and [11] in more details) the user adaptivity is handled by combination of existing (off-line collected) human opinions (user models) in the function of their approximated similarity to the actual user opinions. The goal of the state-transition control is to estimate the “current state”, the actual suitability of the existing user models. Based on the observations (inputs) – the conclusion of the user feedback (the similarity of the user feedback to the existing user models SS_i for all the possible models $\forall i \in [1, N]$) and the previous state S_i (estimation) the state-transition rule base has to estimate the new state values, the next approximation of the vector of the suitability of the existing user models.

The heuristic we would like to implement in our example is very simple. If we already found a suitable model (S_i) and the user feedback is still supporting it (SS_i), we have to keep it even if the user feedback began to support some other models too. If there were no suitable model, but the user feedback began to support one, we have to pick it at once. In case of interpolation-based fuzzy reasoning, the above heuristic can be simply implemented by the following state-transition rule base [10], [11]. For the i^{th} state variable S_i , $i \in [1, N]$ of the state vector S :

$$\text{If } S_i=\text{One} \quad \text{And } SS_i=\text{One} \quad \text{Then } S_i=\text{One} \quad (12.1)$$

$$\text{If } S_i=\text{Zero} \quad \text{And } SS_i=\text{Zero} \quad \text{Then } S_i=\text{Zero} \quad (12.2)$$

$$\text{If } S_i=\text{One} \quad \text{And } SS_i=\text{Zero} \\ \text{And } SS_k=\text{Zero} \quad \text{Then } S_i=\text{One} \quad \forall k \in [1, N], k \quad (12.3)$$

$$\text{If } S_i=\text{Zero} \quad \text{And } SS_i=\text{One} \\ \text{And } S_k=\text{Zero} \quad \text{And } SS_k=\text{Zero} \quad \text{Then } S_i=\text{One} \quad \forall k \in [1, N], k \quad (12.4)$$

$$\text{If } S_i=\text{Zero} \quad \text{And } SS_i=\text{One} \\ \text{And } S_k=\text{One} \quad \text{And } SS_k=\text{One} \quad \text{Then } S_i=\text{Zero} \quad \exists k \in [1, N], k \quad (12.5)$$

where SS_i is the similarity of the user feedback to the i^{th} existing user model $\forall i \in [1, N]$; N is the number of known user models (state variables). The structure of the state-transition rules is similar for all the state variables. *Zero* and *One* are linguistic labels of fuzzy sets (linguistic terms) representing high and low similarity. The interpretations of the *Zero* and *One* fuzzy sets can be different in each S_i, SS_i universes.

Please note that rule base (12) is sparse. It contains the main rules for the following straightforward goals only: Rule (12.1) simply keeps the previously chosen state values in the case if the symptom evaluation also agrees. The rule (12.2) has the opposite meaning, if the state values were not chosen, and moreover the symptom evaluation is also disagrees the state value should be suppressed. The rule (12.3) keeps the already selected state values (previous approximation), even if the symptom evaluation disagrees, if it has no better “idea”. Rules (12.4) and (12.5) have the task of ensuring the relatively quick convergence of the system to the sometimes unstable (changeable) situations, as new state variables which seem to be fit, can be chosen in one step, if there is no previously chosen state, which is still accepted by the symptom evaluation (12.4). (Rule (12.5) has the task to suppress this selection in the case if exists a still acceptable state which has already chosen.) The goal of this heuristic is to gain a relatively quick convergence for the system to fit the opinions of the actual user, if there is no state value high enough to be previously accepted. This quick convergence could be very important in many application areas e.g. in case of an on-line user adaptive selection system introduced in [10], where the user feed-back information needed for the state changes are very limited.

Some state changes of the state-transition control (fuzzy automaton) in the function of the user feedback (SS_1, SS_2) for the two states case (applying the state-transition rule base (12)) are visualised on Fig.5. and Fig.6.

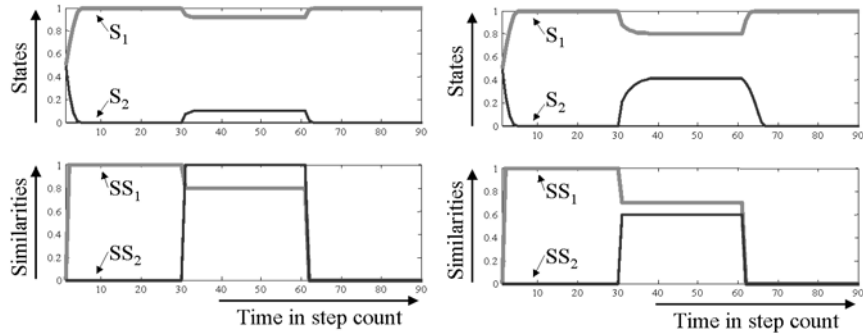


Fig. 4. Do not “pick up” a new state if the previous approximation is still adequate

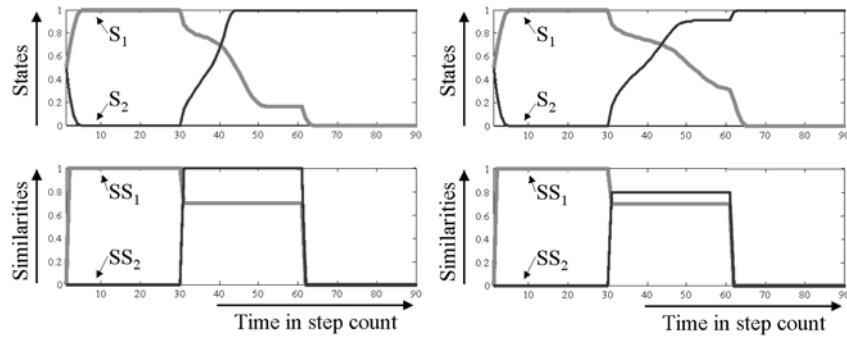


Fig. 5. But “pick it up” if it seems better, or at least as good as the previous was

Counting the rules of the classical (e.g. compositional) fuzzy reasoning for the same strategy we find, that in the two state case the complete rule base needs 16 rules (as we have four observation universes (S_1, SS_1, S_2, SS_2) each with two terms fuzzy partitions ($Zero, One$) - 2^4 rules), while the sparse rule base (12) contains 5 rules only (see table 1 for the state-transition rule base of state S_1). Taking into account that in the proposed behaviour-based control structure a separate rule base is needed for each state variables, the behaviour coordination needs 32 rules, while 10 is enough in case of applying the proposed interpolation-based fuzzy reasoning method. Increasing the number of the state variables the situation became even worse. In case of three state variables (S_1, S_2, S_3) the rate become $3 \cdot 2^6$ ($n \cdot 2^{2^n}$, where n is the number of the states) and $3 \cdot 6$ ($n \cdot (n+3)$) up to the interpolation-based method (see table 2).

Table 1. State-transition rule base of state S_1 in case of two state variables (S_1, S_2) according to rule base (12)

R_{S_1} :	S_1	SS_1	S_2	SS_2	S_1	
1.,	One	One			One	(according to (12.1))
2.,	Zero	Zero			Zero	(according to (12.2))
3.,	One	Zero		Zero	One	(according to (12.3))
4.,	Zero	One	Zero	Zero	One	(according to (12.4))
5.,	Zero	One	One	One	Zero	(according to (12.5))

Table 2. State-transition rule base of state S_1 in case of three state variables (S_1, S_2, S_3) according to rule base (12)

R_{S_1} :	S_1	SS_1	S_2	SS_2	S_3	SS_3	S_1	
1.,	One	One					One	(12.1)
2.,	Zero	Zero					Zero	(12.2)
3.,	One	Zero		Zero		Zero	One	(12.3)
4.,	Zero	One	Zero	Zero	Zero	Zero	One	(12.4)
5.,	Zero	One	One	One			Zero	(12.5)
6.,	Zero	One			One	One	Zero	(12.5)

The exponential rule number “explosion” in case of increasing the number of the input variables makes many heuristic ideas unimplementable and therefore useless. E.g. in the case of the original source of the example application of this paper (introduced in [10]), the behaviour coordination module applied for user adaptive information retrieval system had 4 state variables (one for each emotional models), which makes our simple rule base (12) practically unimplementable as a complete rule base ($4 \cdot 2^8 = 1024$ rules). While our working demonstrational example had only 28 rules thanks to the applied interpolation-based fuzzy reasoning method.

Conclusions

The goal of this paper was to introduce an interpolation-based fuzzy reasoning method, which could be implemented to be simple and quick enough to fit the requirements of real-time direct fuzzy logic control systems. The suggested approximate fuzzy reasoning method based on interpolation in the vague environment of the fuzzy rule base gives an efficient way for designing direct fuzzy logic control applications. The lack of the fuzziness in the conclusion is a disadvantage of the proposed method, but it has no influence in common applications where the next step after the fuzzy reasoning is the defuzzification.

To give some guidelines for interpolation-based fuzzy reasoning rule base design, some highlights of the state-transition rule base of a user adaptive information retrieval system application ([10], [11]) is also introduced in this paper.

The implementation of interpolation-based fuzzy reasoning methods in fuzzy control structures simplifies the task of fuzzy rule base creation. Since the rule base of a fuzzy interpolation-based controller is not necessarily complete, it could contain the most significant fuzzy rules only without risking the chance of having no conclusion for some of the observations. In other words, during the construction of the fuzzy rule base, it is enough to concentrate on the cardinal actions; the “filling” rules (rules could be deduced from the others) could be deliberately omitted. Thus, compared to the classical fuzzy compositional rule of inference, the number of the fuzzy rules needed to be handled during the design process could be dramatically reduced.

The necessity of the complete rule base in many classical fuzzy reasoning methods (e.g. max-min CRI) and hence the exponential rule number “explosion” in case of increasing the number of the input variables makes numerous rule base ideas unimplementable and therefore useless. The application of interpolation-based fuzzy reasoning methods could provide some implementation chances for many of them (see e.g. our simple example in section 3).

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References

- [1] Shepard, D.: A two dimensional interpolation function for irregularly spaced data, Proc. 23rd ACM Internat. Conf., (1968) 517-524
- [2] Klawonn, F.: Fuzzy Sets and Vague Environments, Fuzzy Sets and Systems, 66 (1994) 207-221
- [3] Barnhill, R., Dube, R., Little, F.: Properties of Shepard’s surfaces, Rocky Mountain J. Math. 13 (1991) 365–382
- [4] Kovács, Sz.: New Aspects of Interpolative Reasoning, Proceedings of the 6th. International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, Granada, Spain (1996) 477-482
- [5] Kovács, Sz., Kóczy, L.T.: Approximate Fuzzy Reasoning Based on Interpolation in the Vague Environment of the Fuzzy Rule base as a Practical Alternative of the Classical CRI, Proceedings of the 7th International Fuzzy Systems Association World Congress, Prague, Czech Republic, (1997) 144-149

- [6] Kovács, Sz., Kóczy, L.T.: The use of the concept of vague environment in approximate fuzzy reasoning, *Fuzzy Set Theory and Applications*, Tatra Mountains Mathematical Publications, Mathematical Institute Slovak Academy of Sciences, Bratislava, Slovak Republic, vol.12. (1997) 169-181
- [7] Tikk, D., Joó, I., Kóczy, L. T., Várlaki, P., Moser, B., Gedeon, T. D.: Stability of interpolative fuzzy KH-controllers, *Fuzzy Sets and Systems*, 125, (2002), 105-119
- [8] Tikk, D.: Notes on the approximation rate of fuzzy KH interpolator, *Fuzzy Sets and Systems*, 138(2), September, (2003), 441-453
- [9] Kovács, Sz.: Interpolative Fuzzy Reasoning and Fuzzy Automaton in Adaptive System Applications, *Proceedings of the IIZUKA2000*, 6th International Conference on Soft Computing, October 1-4, Iizuka, Fukuoka, Japan (2000) 777-784
- [10] Kovács, Sz., Kubota, N., Fujii, K., Kóczy, L.T.: Behaviour based techniques in user adaptive Kansei technology, *Proceedings of the VSMM2000*, 6th International Conference on Virtual Systems and Multimedia, October 3-6, Ogaki, Gifu, Japan (2000) 362-369
- [11] Kovács, Sz.: Fuzzy Reasoning and Fuzzy Automata in User Adaptive Emotional and Information Retrieval Systems, *Proceedings of the 2002 IEEE International Conference on Systems, Man and Cybernetics*, October 6-9, Hammamet, Tunisia, 02CH37349C, ISBN: 0-7803-7438-X, WP1N5 (2002) 6
- [12] Kóczy, L. T., Hirota, K.: Interpolative reasoning with insufficient evidence in sparse fuzzy rule bases, *Information Sciences* 71, (1992) 169-201