Critical Analysis of Various Known Methods for Approximate Reasoning in Fuzzy Logic Control

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Abstract:In this paper a rewiev and a critical analysis of various, soft computing based, approximate reasoning methods is given, studied in the Phd thesis of the author. Since Mamdani first implemented a fuzzy logic controller it has also gone trough substantial theoretical developments and serious discussions about place of this method in theory of computational rule of inference. The paper gives a review of these notes.

Keywords:approximate reasoning, FLC

1 Introduction

The applications-based fuzzy compositional rule of inference and fuzzy approximate reasoning includes the thorough investigation of its theoretical basis. As has been mentioned in the introduction, fuzzy applications are specifically task-oriented when choosing a mathematical model. Due to these characteristics, fuzzy systems often apply stochastic instinct-based rules. However, it may be realized later that these rules do not comply with the well-known mathematical basics, or the calculus needs generalization or modification. A number of examples can be listed, pseudo-analysis, for example, which has provided a set of theories based on generalized operation research, that do correspond with theoretical and applied fuzzy systems. [1] contributes some results to this area of research in the first place.

If we examine the activities in the field of fuzzy-based compositional rule, inference and fuzzy approximate reasoning, and fuzzy control system, certain schools and regions come to the forefront in terms of either research or application. Most research work within a region is done by regional fuzzy societies operating under the sponsorship of EUFIT (the European fuzzy society) and the world fuzzy society, IFSA (International Fuzzy System Association).

In connection with European fuzzy schools, it ought to be observed that the world's leading theoretical fuzzy researchers are clustered in these schools. First of all, we need to stress the significance of the Linz School, which has been the meeting point of Europe's and the world's top fuzzy researchers for the past twenty-five years (known as the Linz Seminar, with E.P.Clement as the key figure). The main theme of the Seminar varies each year, but the presentations are set against sound theoretical background, and are of determining importance in the world, in terms of terminology and classification.

Not in order of importance, we shall mention the Slovakian and Czech theoretical fuzzy schools (Mesiar, Hájek), the fuzzy team at Gent University (Kerre, Rubens, De Baets), the Spanish schools, and as well as the German schools. The French school with Dubois and Prade as key figures has been determinant for decades in the field of possibility approximate reasoning. In terms of Hungarian fuzzy research, it is both theory-based, yet we meet much control application and decision making application.

The most important fuzzy school in Serbia and Montenegro is certainly that at the University of Novi Sad, but there are also engineers active in this field (at the Mihajlo Pupin Insitute in Belgrade, at the Subotica Polytechnic, etc.).

It is vital to stress that while the topic has been accepted in practice, it ought to be stressed that there are compromises to be made in order to be applicable, and these will eventually lead to theoretical classification, as well. Schools of the Far East are heavily application-oriented. The U. S. school is among the leading schools in both the fields of theory and application, as Professor Zadeh is still active, and Berkley University remains as one of the leading centers of research.

2 Strict mathematical background or application-orientated approach?

In [2] the equivalence between the study of relations and general system theory has been stressed, pointing out the importance of studying fuzzy relation equations: if a fuzzy relation is considered a system, the composition of a fuzzy relation with a fuzzy set (input) yields a fuzzy set (output), [3]. The fuzzy numbers are used as models in situations, where numbers from the system are only approximately known. The logical interpretation of the computational rule of inference is in a close relationship with the implications and relations [4] (See *Section 4.1.*). From the group of inference mechanisms the generalized modus ponens is the dominant.

Many mathematical models for uncertainty, differing distinctly from probability, have recently been proposed. Models for approximate reasoning in expert systems, with several representations of the IF…THEN rules, and examples for behaviour of the solutions of the computational rule of inference with computer programs can be found in [5].

In [3] the theory of fuzzy relations, namely the solution of fuzzy relation equations (*FRE*) was widely discussed. The theory of such equations was initiated by Sanchez in 1976., [6], but the book [3] has a dual purpose: first, to present an rigorous, thorough and complete theory of the solution of fuzzy relation equations, and second, to describe its applications, especially in the knowledge-based systems.

In [7] the generalized modus ponens is confronted with the triangular fuzzy data model. The possible representations of non-fuzzy and fuzzy SISO systems are discussed extensively. Five families of implications, (defined in the paper) and correspondent families of generalized modus ponens inference rules are discussed. It is shown that the conclusions obtained with these inference rules and triangular fuzzy numbers can be expressed in an exact analytical way. In the fifth Chapter of the PhD thesis [1] these investigations are extended with further cases, using *g*-calculus and pseudo-operations.

In [8] a multiple-rule, generalized modus ponens has been considered, based on compositional rule of inference and residuated implication. This system satisfies the "basic requirement for fuzzy reasoning", proposed by Turksen and Tian, [9].

In [10] three typical scenarios were discussed: intersection of fuzzy sets, transitivity of fuzzy relations, and addition of fuzzy numbers, where the minimum should be replaced by some more appropriate triangular norm.

Since Mamdani first implemented a fuzzy logic controller, [11], the basic principle of fuzzy logic controllers has found applications in a variety of industrial uses, but has also gone trough substantial theoretical developments and serious discussions about place of this method in theory of computational rule of inference. "Mamdani controllers have always been a source of controversy: its proponents praise its easy implementation and its proven practicality while its opponents criticize its unorthodox rule modeling", [12].

In the original Mamdani's algorithm the *min* and *max* operators have been used as intersection and union operations for fuzzy sets, and for the implication in the rule of inference. The first generalizations of the Mamdani model were those based on generalized t-operators [13], and from this time a large group of several operators can be found in engineering applications [14].

"There are many papers describing various properties of fuzzy operations and relations with respect to different tasks to be solved. For example, for engineering applications the cause and effect relation between the propositions of the if-then statement is usually assumed. It has been reported several times that if we take into account the causality constraint, the best results in engineering applications are delivered with the minimum and algebraic product operations. However, it should be admitted that despite the fact that many fuzzy relations do not fulfil the causality condition, some of them may deliver quite reasonable results in engineering and other areas also, e.g., in decision making. "[15]

Based on experiences in engineering applications an axiom system has been constructed, granting the basic conditions supposed for the approximate reasoning for a fuzzy rule base system and inference mechanism (see *Section 4.3.4* in [1]). Fuzzy controllers with conditionally firing rule (like the Mamdani model) have been investigated in [16] in the context of this axiom system. Situated between the theses of the [1], these axioms declare the expectations of the mention approximate reasoning systems, and it becomes clear to what degree they satisfy or violate this axiom system. Nevertheless it is obvious, that using of mathematically established implications (for example the Gödel implication) violates first of all the axiom (out3). ([17], Section 4.1.2.), [18], Section 2.3.6.).

From the application point of view two most important and most widely used fuzzy controllers currently are the Mamdani-type controller [11], [19], described in *Section 4.3.2.* in [1], and the Takagi-Sugeno-type controller [20].

The Takagi-Sugeno fuzzy modeling is a technique to describe a nonlinear dynamic system using local linearized models and crisp values in the output space. The Takagi-Sugeno model prefers the rules in rule base in the form

$$\text{if } x \text{ is } A_i \text{ then } y=f_i(x),$$

and the inference has a built-in defuzzification:

$$F_{TS}(x)=\frac{\sum_{i=1}^{n} A_i(x)\cdot f_i(x)}{\sum_{i=1}^{n} A_i(x)}.$$

For a Takagi-Sugeno model with the normal fuzzy subsets A_i, and rules in the form "if x is A_i then $y=y_i$", and with Borel–measurable membership functions with property $\sum_{i=1}^{n} A_i(x) > 0$ there exists a Mamdani controller corresponding input-output function of the Mamdani model coincides with F_{TS} .[21], *Proposition 13.16.*).

3 Soft Computing

If we can represent the fuzzy logic system as a feed-forward network, we can use the idea of back-propagation to train them, and combine neural network theory and genetic algorithm theory with fuzzy possibilities. This complex application of *Soft computing* technologies leads to the adaptive fuzzy systems [22], [23], and started the theoretical investigations about pseudo-analysis as a mathematical base for soft computing, [24].

The main module of the adaptive fuzzy systems is the module for learning fuzzy rules. Learning fuzzy rules can take place either before the inference machine starts the reasoning process, or during the fuzzy inference process if the following phases of a fuzzy system design are considered [25]:

(i) Identifying the problem and choosing the type of fuzzy system which best suits the problem requirements.

(ii) Defining the input and output variables their membership functions.

(iii) Articulating the set of heuristic fuzzy rules.

(iv) Choosing the fuzzy inference method, fuzzification, and defuzzification methods.

(v) Experimenting with the fuzzy system prototype:

- drawing the goal function between input and output fuzzy variables,
- changing membership functions and fuzzy rules if necessary (for example with linear rule interpolation, [26].

With the rapid increase of computation, machine learning methods such as genetic algorithms are gaining more attention as system design tools. In [27] a systematic approach for designing a multistage fuzzy control system can be found. For the large rule base systems hierarchical rule base construction has been applied (see [28], [29]).

Conclusions

Considering the application-oriented choice of the fuzzy model in engineering systems, it can be stated, that new adaptive fuzzy operators and approximate reasoning methods are needed in FLC-s, but at all times based on strict mathematical theoretical background.

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