Positioning Sensor for Mobile Robot

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Abstract: *The experiments with one-wheel odometric system for mobile robots are reported. The necessary mathematical description of the system is given at the beginning, after that the positioning results of the mobile robot traveling along two kinds of the paths are given: the one consisting of many turns and the other consisting mostly of straight sections.*

Keywords: *Mobile Robots, Position Sensor, Odometry*

I INTRODUCTION

Odometry still seems to be effective and relatively cheap method of providing mobile robot with current position. Absolute positioning methods [1] usually provide with absolute position only on certain points of the working space: it is then some relative measuring method which is expected to provide position from one absolute position to another. Odometry is a reasonable choice for such a purpose.
The application of additional measuring wheels (AMW) represent accurate method of calculation of the current position. These wheels are normally mounted on the left and right side of the robot. It has been reported [2] that the most common of the systematic error sources caused by AMW are unequal wheel diameters and inaccurate effective wheelbase.
We propose the method which avoids these kinds of errors.

II ONE-WHEEL POSITIONING SENSOR

The sensor may be observed in Figure 1. It consists of optical encoder for the measurement of the traveled distance, traveled distance encoder and optical encoder for measurement of the robot's current steering angle, steering encoder. The construction of the traveled distance encoder is unique: it is based on the reflection of the emitted light from the mask, glued on the measurement wheel, as opposed to transparent masks used in the common encoders.
As the robot moves on, the additional wheel, mounted eccentrically to the vertical axis of the steering encoder, gradually aligns with the direction of the movement.
Mathematical description of the displacement is given in the following two sections and it is divided into two cases: the measurement wheel is supposed to be (1) or not to be (2) aligned with the direction of the movement.

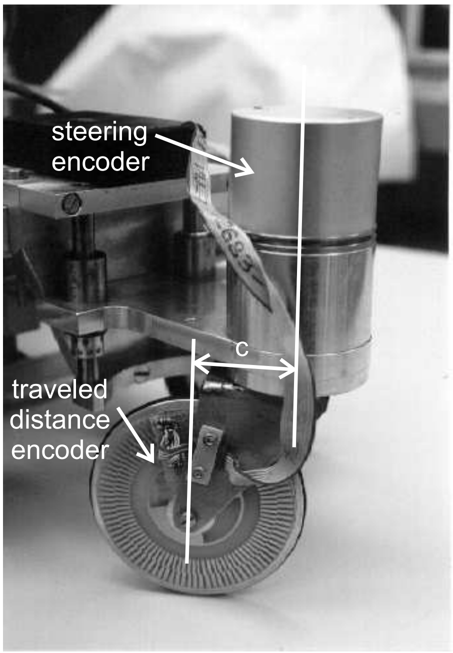

Figure 1: *One-wheel positioning sensor mounted in front of the robot.*

III ALIGNED MEASUREMENT WHEEL $\mathbf{OPS_A}$

The situation in which the robot moves along circular arc is drawn on Figure 2. T_1 is the reference point of the robot for which all the position calculations are made, B is the position of the steering encoder which measures steering angle β, K (K_1) is circular arc traveled along by robot's reference point (distance wheel), A is the contact point of the distance wheel and c is the eccentricity of two orthogonal axles (one being of distance wheel, the other of steering encoder). The derivation of $\Delta\Theta$ and Δl, which denote increment in orientation and arc length traveled along by reference point is based on right and also similar triangles $\triangle S'T_1G \sim \triangle BAG \sim \triangle S'AE \sim \triangle BT_1E$. Cur-

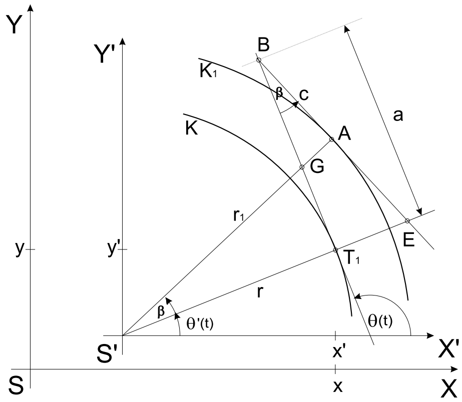

Figure 2: *Geometry of* $\mathrm{OPS_A}$

vatures[1] K and K_1 are determined by

$$K = \frac{\sin(\beta)}{a\cos(\beta) - c}$$

$$K_1 = \frac{K\cos(\beta)}{1 + cK\sin(\beta)}$$

After the curvatures of both circular arcs are known, $\Delta\Theta$ and Δl are given by (reader should refer to [3] for the detailed procedure)

$$\Delta\Theta = \frac{\Delta l_1 K\cos(\beta)}{1 + cK\sin(\beta)} \tag{1}$$

$$\Delta l = \frac{\Delta l_1 \cos(\beta)}{1 + cK\sin(\beta)} \tag{2}$$

where Δl_1 is the distance, measured by distance encoder. The above equations are valid only for aligned distance wheel, what is not really the case all the time. It is not aligned nor at the beginning nor during the movement. Control of the (differential) robot is realised by driving on tiny portions of circular arcs from one control position to the other, even if the robot is said to be moving linearly. As a consequence, the AMW

[1] curvature is used throughout the article, since we consider it is limited - the robot does not turn-on-spot

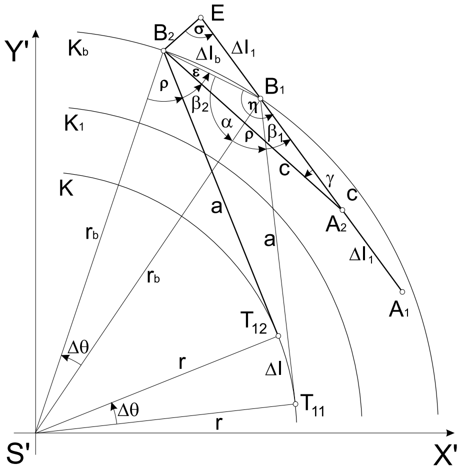

Figure 3: *Geometry of* $\mathrm{OPS_N}$

is repetitively not aligned what causes error in orientation Θ. It is interesting to know then, how does the orientation of the distance wheel approach its balanced orientation. It was shown [3], for example, that the distance $\approx 0.5c$ was needed to reduce the nonalignment angle for a half, when driving on a line (from $\beta_{INIT} = \pi/2$, what means perpendicular wheel to the path, to $\beta = \pi/4$).

IV NONALIGNED MEASUREMENT WHEEL $\mathbf{OPS_N}$

Again, let us assume that the robot moves along a circular arc. The illustration of such a movement can be seen on Figure 3. While the reference point T_{11} moves along the circular arc with the curvature K into point T_{12}, point of the steering encoder B_1 moves along the arc with the curvature K_b into point B_2 and, similarly, contact point of the distance wheel moves from A_1 into A_2. If the distance wheel was in its balanced orientation Θ, A_i points would lie on arc with, say, curvature K_1. Suppose we make an assumption that the movement is small enough to approximate that A_2 lies on the line segment $\overline{A_1B_1}$ while displacement Δl_1 is measured by the distance encoder. Δl_b and Δl are circular arcs while Δl_1 is then a straight line. We place the point E in such a way that isosceles triangle $\triangle A_2B_2E$ is created. Then in turn, the following angles and variables are determined.

- ρ is given by $\rho = \arccos(a/r_b)$ considering the $\triangle S'T_{11}B_1$ and $\triangle S'T_{12}B_2$
- α and σ (from $\triangle S'B_1B_2$ and $\triangle A_2B_2E$)
- angles ε, η, γ and edges of $\triangle A_2B_2B_1$

Sine Theorem in $\triangle A_2B_2B_1$ gives us the difference equation for $\Delta\Theta$ from which we obtain the differential equation for $d\Theta$. Similar procedure may be applied on $\triangle B_1B_2E$ to obtain differential equation for dl_1, also. Integration of both equations leads us to the expressions for $\Delta\Theta$

$$\Delta\Theta = \frac{cK_b}{\sqrt{1-cK_b{}^2}} \ln \frac{\Phi_+(\beta_2)\,\Phi_-(\beta_1)}{\Phi_-(\beta_2)\,\Phi_+(\beta_1)} \tag{3}$$

where

$$\Phi_-(\beta_i) = (1+cK_b)\tan\left(\frac{\beta_i+\rho}{2}\right) - \sqrt{1-cK_b{}^2}$$

$$\Phi_+(\beta_i) = (1+cK_b)\tan\left(\frac{\beta_i+\rho}{2}\right) + \sqrt{1-cK_b{}^2}$$

and Δl_1

$$\Delta l_1 = c\ln\left(\frac{\cos(\beta_1+\rho)-cK_b}{\cos(\beta_2+\rho)-cK_b}\right) \tag{4}$$

The complete procedure may be found in [4]. The equations described above

may be used only with different angles β_1 and β_2, that is, when the distance wheel is really nonaligned.
The use of OPS_N consists of the following steps:

- β_2 and Δl_1 are read from steering and distance encoder
- if β_2 matches the angle β_1 from the previous measurement, the OPS_A is used (equations 1, 2), otherwise we proceed with the next step
- cK_b and ρ are calculated
- $\Delta\Theta$ is calculated
- Δl using (5) is calculated

$$\Delta l = \Delta\Theta \sqrt{\left(\frac{c}{cK_b}\right)^2 - a^2} \qquad (5)$$

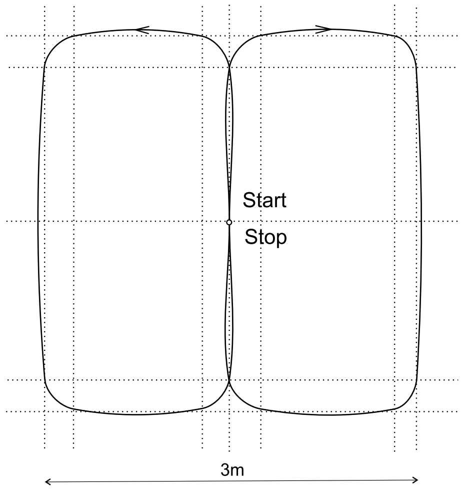

Figure 4: *'Eight-turn' path.*

V EXPERIMENTAL RESULTS

Two types of experiments were conducted in order to test our positional sensor: in the first, Figure 4, the robot was programmed to move along 'eight-shape' path (to test the conditions with many turns); in the second, Figure 6 the robot traveled through the long and narrow corridor (to test robot's ability to maintain given direction). The length of the first path was around 20 meters while the second was considerably longer: 120m. Both paths were closed, so it could be seen at a glance how much the robot's final position differed from the starting point. In Figure 5 we may observe the positional error at the final position. The error is something bigger in longitudinal, Y, than in lateral X, direction, yet it is

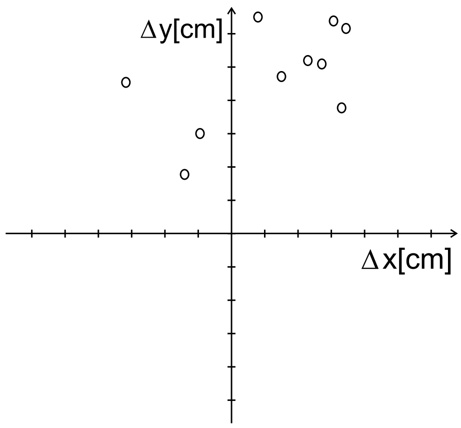

Figure 5: *Errors in position after 'eight-turn' travels.*

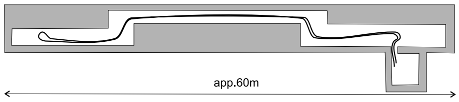

Figure 6: *Path trough the corridor.*

still quite small, even for the category of robots with additional measurement wheels. We ascribe such an error also to the symmetrical structure of the path [2].

The main problem with such a long path as in the second case was determination of the offset angle β_0 of the directional encoder which measures the orientation of the robot. Inaccurate β_0 causes the rotation of the calculated paths, robot's journey prematurely ends in the wall. Our choices for β_0 successively decreased from the value $196,708°$ to $196,617°$, until the robot for $\beta_o = 196,608°$ managed to reenter the laboratory, after app.120m of the traversed path. The positional errors of the four such traversals are given in the following table:

Path	$\Delta x[m]$	$\Delta y[m]$
1.	-0,101	-0,309
2.	0,079	0,430
3.	0,024	0,261
4.	0,068	0,372

Conclusion

We presented the experimental results of the odometric system, based on additional measurement wheel, which consists of absolute encoder and indirect optical relative encoder. The first one measures direction and the second one the traveled path of the robot. The mathematical description of the system is not explained in details (look in [4]). The experiments showed the accuracy of the proposed odometric system is quite comparable to the classical solution with two additional measurement wheels, one on each side of the robot. However, the values of four parameters: β_0, c, r_1, a, which have to be set are not acquired easily: some procedure is needed to determine their values systematically. On the other side, the advantage of the tested odometric system lies in the fact it can be mounted elsewhere on the robot; the user is far less restricted than with two-wheel system. It is also important to note that the initial nonalignment of the distance wheel with the initial orientation of the robot does not represent a problem: the mathematical description given in the section where nonaligned distance wheel was considered, proved to hold also in the case of major initial nonalignment.

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