

# Motion Control Simulation with Special Fuzzy Operators

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**Abstract:** *Decision making in FLC system, based on distance-based operators and fuzzy approximate reasoning model will be discussed. Simulation results will be analyzed for a first order problem, usually manifested in motion control problems, where the modification of the boundary conditions for distance based operator group leads to make wider family of applicable operators in FLC systems. Distance-based operators, as evolutionary operators group enable us to verify the effect of different applied fuzzy operators in decision making process in different cells of a complex system, just changing one parameter by operators.*

**Keywords:** *FLC, uninorms, approximate reasoning*

## I INTRODUCTION

Experimental and application results in recent time have proven that in different systems, where fuzzy can be implemented, it is allowed to apply and test different fuzzy models, to get better system behavior. The differences between fuzzy models lie in the choice of applied operators, or if it is a control problem, in different approximate reasoning methods [1]. Considering, that systems in the real world today are getting more and more complex, it is clear, that the same fuzzy environment cannot be equally powerful in each cell of the system. Motion control problem is one of those complex problems, investigated in many sources and from many points of view. One of the possible ways to the right, planned motion is measuring of one of the environment states, comparing it with the desired state,

and based on that information making decision for the further steps and actions [13]. Summarized decision making results for a group of states of the system can be base for a higher level in a hierarchical decision making system, which is necessary in the high-complex system [4].

Instead of the usually applied minimum and maximum operators pair in fuzzy logic control (FLC) systems there are more and more new operators in the scope of experts. Uninorms, which are for some arguments minimum-like, and for the rest of arguments maximum-like, allow decision making closer to the human mind ([1],[9],[11]). Evolutionary operators group admit, that the domain of the arguments, where minimum-like and where the maximum-like operator use to be applied, can be modified depending on a parameter. It enable us to verify the effect of different applied

fuzzy operators in decision making process in different cells of a complex system, just changing one parameter by operators.

In further sections first of all modified distance-based operators will be explained, which are evolutionary-type operators. Following that decision making in FLC system, based on fuzzy approximate reasoning model will be discussed. At the end simulation results will be analyzed for a first order problem, usually manifested in motion control problems, where the modification of the boundary conditions for distance based operator group leads to make wider family of applicable operators in FLC systems.

## II DISTANCE BASED OPERATORS

The modified distance-based operators on the unit square  $(x, y) \in [0,1]^2$  ([6], [7]) can be expressed by means of the min and max operators as follows.

The *maximum distance minimum operator* with respect to  $e \in [0,1[$  is defined as

$$\max_e^{\min} = \begin{cases} \max(x, y), & \text{if } y > 2e - x \\ \min(x, y), & \text{if } y < 2e - x \\ \min(x, y), & \text{if } y = 2e - x \end{cases}$$

Further modification of this operator is, if on the boundary lines  $(0,y)$  and  $(x,0)$  we define maximum distance base minimum value as  $\max(x,y)$ . Let as denote this operator as *boundary modified maximum distance minimum operator*.

The *minimum distance minimum operator* with respect to  $e \in [0,1[$  is defined as

$$\min_e^{\min} = \begin{cases} \min(x, y), & \text{if } y > 2e - x \\ \max(x, y), & \text{if } y < 2e - x \\ \min(x, y), & \text{if } y = 2e - x \end{cases}$$

The maximum distance maximum operator with respect to  $e \in [0,1[$  is defined as

$$\max_e^{\max} = \begin{cases} \max(x, y), & \text{if } y > 2e - x \\ \min(x, y), & \text{if } y < 2e - x \\ \max(x, y), & \text{if } y = 2e - x \end{cases}$$

the minimum distance maximum operator with respect to  $e \in [0,1[$  is defined as

$$\min_e^{\max} = \begin{cases} \min(x, y), & \text{if } y > 2e - x \\ \max(x, y), & \text{if } y < 2e - x \\ \max(x, y), & \text{if } y = 2e - x \end{cases}$$

The maximum distance minimum operators and the modified group, except on the border of the original area, are from disjunctive operators group, and the minimum distance maximum operators are from conjunctive operators group [8].

Changing the parameter  $e$  from 0 to 1, the maximum distance minimum operator evaluated from the maximum on the unit square in the minimum on the hole unit square.

## III DECISION MAKING AND APPROXIMATE REASONING IN FLC BASED ON DISTANCE BASED OPERATORS

In Mamadani-based FLC sources it was suggested to represent the  $i^{\text{th}}$  rule if  $x$  is  $A_i$  then  $y$  is  $B_i$

as a connection (for example as a t-norm,  $T(A_i, B_i)$  or as  $\min(A_i, B_i)$ ) between rule premise:  $x$  is  $A$  and rule consequence:  $y$  is  $B$ .

The strict modus ponens in FLC is replaced with the expectation: let be  $B' \supset B$ , where  $B'$  is a cut of  $B$ . That is the Generalized Modus Ponens (GMP), in which the main point is, that the inference  $y$  is  $B'$  is obtained when the propositions are:

- the  $i^{th}$  rule from the rule system of  $n$  rules: if  $x$  is  $A_i$  then  $y$  is  $B_i$ ,
- and the system input  $x$  is  $A'$ .

Usually the general rule consequence for one rule from the  $i$ -th rule base system in fuzzy logic systems, based on the mathematical model of decision making and approximate reasoning, which is compositional rule of inference ([2],[3],[5]), is obtained by

$$B_i'(y) = \sup_{x \in X} (OPDis1(A'(x), OPDis2(A_i(x), B(y))))$$

The connection  $OPDis1$  and  $OPDis2$  are generally defined, and they can be some type of fuzzy disjunctive operators [2], [3].

Using the same disjunctive operator  $OPDis$  in both of cases, and based on operator properties, from the above expression follows

$$B_i'(y) = \left( OPDis \left( \sup_{x \in X} OPDis(A_i(x), A'(x)), B(y) \right) \right)$$

The consequence (rule output) is given with a fuzzy set  $B'(y)$ , which is derived from rule consequence  $B(y)$ , as a cut of the  $B(y)$ . This cut,

$$DOF = \sup_{x \in X} OPDis(A_i(x), A'(x))$$

is the generalized degree of firing level of the rule, considering actual rule base input  $A'(x)$ , and usually depends on the covering over  $A(x)$  and  $A'(x)$ . But first of all it depends on the  $sup$  of the membership function of  $OPDis(A'(x), A(x))$ .

Rule base output,  $B'_{out}$  is an aggregation of all rule consequences  $B_i'(y)$  from the rule base. As aggregation operator a conjunctive fuzzy operator is usually used.

$$B'_{out}(y) = OPCon(B_n'(y), OPCon(B_{n-1}'(y), OPCon(\dots, OPConS(B_2'(y), B_1'(y)))))$$

The crisp FLC output  $y_{out}$  is constructed as a crisp value calculated with a defuzzification method.

It can be concluded, that in approximate reasoning the  $(OPDis, OPCon)$  pair of operators are used, and can be chosen from the group of distance based operators.

Moreover, the distance based operators are parameterized by the parameter  $e$ , therefore the FLC has global, optional, variables  $(OPDis, OPCon, e)$ , where  $OPDis$  is the operator applied by GMP, and the  $OPCon$  is the aggregation operator for the calculation of the  $B'_{out}$ . The neutral element of the  $OPDis$  operator is parameter  $e$ , and the neutral element of the  $OPCon$  operator is parameter  $1-e$ .

#### IV SIMULATION RESULTS

The simulation system was built in the MATLAB-SIMULINK environment and the program (S-function) of the FLC model in C programming language. Graphical interface in Delphi environment enable as:

- choosing of the  $OPDis$  and  $OPCon$  operators from the group of the distance based and modified distance based operators,
- sliding of the parameter  $e$  of the distance based operators (in the interval  $]0,1]$ ),
- sliding of the center of the fuzzy sets of rule premises and rule consequences in rules of the fuzzy rule base.

Details can be found in [12].

Considering, that the simulation model was a model of a first order problem, and the desired state was a step function, in [12] it was concluded, that the

$$(OPDis, OPCOn, 0.5) = (\max_{0.5}^{\min}, \max_{0.5}^{\min}, 0.5)$$

choice, by the uniform, well known arrangement of the membership functions of the rule premisses and rule consequences in the rule base system leads to the best results. The step function as the FLC output reaches the intensity 1, and the system stay stable after 0,5 seconds.

In the case of

$$(OPDis, OPCOn, e) = (\max_e^{\min}, \min_{1-e}^{\max}, e)$$

( $e \neq 0,5$ ), the conclusions were: the step function as the FLC output reaches the intensity 1, and the system portrays irregularities periodically (Figure 1).

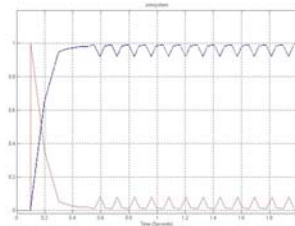


Figure 1

case of  $(\max_e^{\min}, \min_{1-e}^{\max}, e), (e \neq 0,5)$

If the boundary modified maximum distance minimum operator replace the non-modified maximum distance minimum operator in the construction  $(\max_e^{\min}, \min_{1-e}^{\max}, e), (e \neq 0,5)$  the periodically irregularities are eliminated, and the FLC output has the form as on Figure 2.

### Conclusion

Evolutionary operators group, in this case distance based and modified distance based operators group admit, that the domain of the arguments, where minimum-like and where the maximum-like operator use to be applied, can be modified depending on the parameter  $e$ , which is the neutral

element of those operators. It enables us to verify the effect of different applied fuzzy operators in decision making process in different cells of a complex system, changing one or more parameters by operators. Boundary modified distance based operator in the construction of the FLC model in some cases contributes to the better behavior of the system. It motivates to complete the possible used families of the operators in FLC with non-wide spread ones. Motion control problem is one of complex problems, where the wide set of applied operators group admits the verification of the effect of different applied fuzzy operators in decision making process in different cells of that complex system.

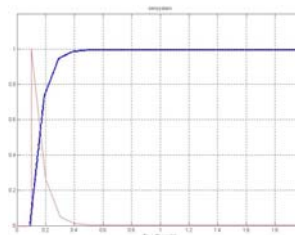


Figure 2

Simulation results with the boundary modified maximum distance minimum operator

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### References

- [1] Bellmann, R. E., Zadeh., L. A., (1977), *Local and fuzzy logic*,

- Modern Uses of Multiple-Valued Logic, edited by Dunn, J. M., Epstein, G., Reidel Publ., Dordrecht, The Netherlands, pp. 103-165
- [2] De Baets, B., Fodor, J., (1999), *Residual operators of uninorms.*, Soft Computing 3, (1999), pp. 89-100
- [3] Fodor, J., Rubens, M., (1994), *Fuzzy Preference Modeling and Multi-criteria Decision Support.* Kluwer Academic Pub., 1994
- [4] Kóczy, T. L., Hirota, K., (1993), *Approximate reasoning by Linear Rule Interpolation and General Approximation*, Int. Jour. Of Approximate reasoning, 9, pp. 197-225
- [5] Klement, E. P., Mesiar, R, Pap, E., *'Triangular Norms'*, Kluwer Academic Publishers, 2000, ISBN 0-7923-6416-3
- [6] Rudas, I., *Absorbing-Norms*, in Proceedings of the 3<sup>rd</sup> International Symposium of Hungarian Researchers on Computational Intelligence, Budapest, Nov. 2002, pp. 25-43
- [7] Rudas, Imre; Kaynak, O., *New Types of Generalized Operations*, Computational Intelligence, Soft Computing and Fuzzy-Neuro Integration with Applications, Springer NATO ASI Series. Series F, Computer and Systems Sciences, Vol. 192. 1998. (O. Kaynak, L. A. Zadeh, B. Turksen, I. J. Rudas editors), pp. 128-156
- [8] Takacs, M., *Approximate Reasoning in Fuzzy Systems Based on Pseudo-Analysis*, Phd Thesis, Univ. of Novi Sad, 2004
- [9] Zadeh, L. A., *A Theory of approximate reasoning*, In Hayes, J., and editors, *Machine Intelligence*, Vol. 9, Halstead Press, New York, 1979., pp. 149-194
- [11] Zimmermann, H. J., (1991), *Fuzzy Sets, Decision Making and Expert Systems.* Kluwer, Boston, 1991
- [12] Takacs, M. Baky, Zs., *Parameter-based Program-interface for a FLC Simulation System*, Proc. Of the SACI 2005 Symposium, Timisoara.
- [13] Yager, R. R., Filev, D. P.: *Essential of Fuzzy Modeling and Control*, Book, New York/John Wiles and Sons Inc./, 1994