# An Algorithm for Kinematic Calibration of Robot Arms 

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#### Abstract

An algorithm for the kinematic calibration of a robot arm is presented. The trajectory, described in the work space, of given point of the last link of a robot is recorded by means of two television cameras. From the data obtained from a number of couples of frames, if the joint position of the robot arm for each couple of frames is measured from the feedback control system, it is possible to compute the Denavit and Hartemberg parameters. A couple of television cameras is employed to obtain a stereoscopic vision. The method has to be linked with a camera calibration technique so that the calibration of a robot arm and of the vision system can be obtained in the same time. The tuning of the technique is still in progress as, presently, allows a precision of little less than $1 \%$.


Keywords: Robot kinematic calibration, Vision system

## I INTRODUCTION

Among the characteristics that define the performances of a robot the most important can be considered the repeatability and the accuracy. Generally, both these characteristics depend on factors like backlashes, load variability, positioning and zero putting errors, limits of the transducers, dimensional errors, and so on. The last sources of error essentially depend on the correct evaluation of the Denavit and Hartemberg parameters. Hence, some of the sources of error can be limited by means of the cinematic calibration.
Basically, by the cinematic calibration it is assumed that if the error in the positioning of the robot's end effector is evaluated in some points of the
working space, by means of these errors evaluation it is possible to predict the error in any other position thus offset it.
In few words, the main aim of the technique showed in this paper is to obtain precise evaluations of those Denavit-Hartenberg parameters that represent, for each of the links, the length, the torsion and the offset.

## II THE CALIBRATION TECHNIQUE

This calibration technique essentially consists in the following steps:
I The end-effector is located in an even position in the work space;
II A vision system acquires and records the robot's image and gives the coordinates of an assigned
point of the end-effector, expressed in pixels in the image plane.
III By means of a suitable camera model, it is possible to find a relation between these coordinates expressed in pixels, and the coordinates of the assigned point of the end-effector in the world (Cartesian) frame.
IV By means of the servomotor position transducers, the values of the joint position parameters are recorded for that end-effector position in the work space.
In this way, for each of the camera images, the following arrays are obtained:

$$
\left(\begin{array}{c}
X_{i} \\
Y_{i} \\
Z_{i}
\end{array}\right), \quad\left(\begin{array}{c}
\theta_{1, i} \\
\theta_{2, i} \\
\theta_{3, i}
\end{array}\right)
$$

where: $i=1, \ldots, N$, and $N$ is the number of acquired camera images (frames).
If the coordinates in the working space and the joint parameters are known, it's possible to write the direct kinematics equations in which the unknown are those DenavitHartenberg parameters that differ from the joint parameters; thus these Denavit-Hartenberg parameters represent the unknown of the kinematic calibration problem.
The expression of these equations is obtained starting from the transform matrix (homogeneous coordinates) that allows to transform the coordinates in the frame $i$ to the coordinates in the frame $i-1$ :

$$
{ }^{i-1} A_{i}=\left[\begin{array}{cccc}
C \vartheta_{i} & -\mathrm{C} \alpha_{i} \cdot \mathrm{~S} \vartheta_{\mathrm{i}} & \mathrm{~S} \alpha_{i} \cdot \mathrm{~S} \vartheta_{\mathrm{i}} & \mathrm{a}_{\mathrm{i}} \cdot \mathrm{C} \vartheta_{\mathrm{i}}  \tag{2}\\
\mathrm{~S} \vartheta_{i} & \mathrm{C} \alpha_{\mathrm{i}} \cdot \mathrm{C} \vartheta_{\mathrm{i}} & -\mathrm{S} \mathrm{\alpha}_{i} \cdot \mathrm{C} \vartheta_{i} & \mathrm{a}_{\mathrm{i}} \cdot \mathrm{~S} \vartheta_{\mathrm{i}} \\
0 & \mathrm{~S} \alpha_{\mathrm{i}} & \mathrm{C} \alpha_{\mathrm{i}} & d_{i} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

By means of such matrixes it is possible to obtain the transform matrix that allows to obtain the coordinates in the frame 0 (the fixed one) from those in frame n (the one of the last link):

$$
{ }^{0} \mathrm{~T}_{\mathrm{n}}={ }^{0} \mathrm{~A}_{1} \cdot{ }^{1} \mathrm{~A}_{2}: \ldots \cdot \cdot{ }^{\mathrm{n}-1} \mathrm{~A}_{\mathrm{n}}
$$

As for an example, if we consider a generic 3 axes revolute (anthropomorphic) robot arm, we'll obtain an equation that contains 9 constant kinematic parameters and 3 variable parameters $\left(\theta_{1}, \theta_{2}, \theta_{3}\right)$.
So, the vector:
$\pi_{\mathrm{DH}}=\left(\begin{array}{c}\mathrm{a}_{1} \\ \mathrm{a}_{2} \\ \mathrm{a}_{3} \\ \mathrm{~d}_{1} \\ d_{2} \\ d_{3} \\ \alpha_{1} \\ \alpha_{2} \\ \alpha_{3}\end{array}\right)$
represents the unknown of the kinematic calibration problem.
Said:
$\Theta=\left(\begin{array}{l}\theta_{1} \\ \theta_{2} \\ \theta_{3}\end{array}\right)$
the direct kinematics equation for this manipulator can be written as:
$w=t_{4}\left(\pi_{D H}, \Theta\right)$
where $w$ is the position vector in the first frame and is the fourth row of the Denavit-Hartenberg transform matrix. In eq. (5) it clearly appears that the position depends on the joint parameters and on the others DenavitHartenberg parameters. The eq. (5) can be also seen as a system of 3 equations (in Cartesian coordinates) with 9 unknowns: the elements of vector $\pi_{D H}$.

Obviously, it's impossible to solve this system of equations, but it's possible to use more camera images taken for different end-effector positions:

$$
\left\{\begin{array}{l}
t_{4}\left(\pi_{D H}, \Theta^{1}\right)=w_{1}  \tag{6}\\
t_{4}\left(\pi_{D H}, \Theta^{2}\right)=w_{2} \\
\ldots \ldots . . . . . . . . . . . . . . . . . . . . . ~
\end{array}\right.
$$

with $N \geq 9$.
As, for each of the camera images the unknown Denavit-Hartemberg parameters are the same, equations (6) represent a system of $N$ non linear equations in 9 unknowns. This system can be numerically solved by means of a minimum square technique.
It's known at a minimum square problem can be formulated as follows: given the equation (5), find the solutions that minimize the expression:
$\int_{D_{\Theta}}\left|t_{4}\left(\pi_{D H}, \Theta\right)-w\right|^{2} \cdot d \Theta$
This method can be simplified by substituting the integrals with summations, thus it must be computed the vector that minimize the expression:

$$
\begin{equation*}
\sum_{i=1}^{N}\left|t_{4}\left(\pi_{D H}, \Theta^{i}\right)-w_{i}\right|^{2} \tag{8}
\end{equation*}
$$

If we formulate the problem in this way, the higher is the number of images that have been taken (hence the more are the known parameters), the more accurate will be the solution, so it's necessary to take a number of pictures.

## III PROSPECTIVE TRANSFORM AND CAMERA MODEL

It's useful to remember that by means of a perspective transform it's possible to associate a point in the geometric
space to a point in a plane, that will be called "image plane"; this will be made by using a scale factor that depends on the distance between the point itself and the image plane.
Let's consider Fig. 1: the position of point P in the frame $\mathrm{O}, \mathrm{x}, \mathrm{y}, \mathrm{z}$ is given by the vector $w$, while the same position in the frame $\Omega, \xi, \eta, \zeta$ is given by vector $w_{r}$ and the image plane is indicated with R ; this last, for the sake of simplicity, is supposed to be coincident with the plane $\xi, \eta$.


Figure 1
The vectors above are joined by the equation:
$\left\{\begin{array}{c}w_{r, x} \\ w_{r, y} \\ w_{r, z} \\ s f\end{array}\right\}=\left[\begin{array}{cccc}R_{11} & R_{12} & R_{13} & t_{\xi} \\ R_{21} & R_{22} & R_{23} & t_{n} \\ R_{31} & R_{32} & R_{33} & t_{\zeta} \\ 0 & 0 & 0 & s f\end{array}\right]=\left\{\begin{array}{c}w_{x} \\ w_{y} \\ w_{z} \\ \text { sf }\end{array}\right\}$
where sf is the scale factor; more concisely eq. (9) can be written as follows:
$\widetilde{w}_{r}=T \cdot \widetilde{w}$
where the "tilde" indicates that the vectors are expressed in homogeneous coordinates.
The matrix T is a generic transformation matrix that is structured according to the following template:


The fourth row of matrix [T] contains three zeros; as for these last by means of the prospectic transform three values, generally different by zero, will be determined.
The scale factor will almost always be 1 and the perspective part will be all zeros except when modelling cameras. Let's consider, now, Fig.2: the vector $w^{*}$ represents the projection of vector $w_{r}$ on the plane $\xi, \eta$.


Figure 2
The coordinates of point P in the image plane can be obtained from the vector $w_{r}$ : in fact, these coordinates are the coordinates of $w^{*}$, that can be obtained as follows.
Let's consider the matrix R:

$$
R=\left[\begin{array}{l}
\hat{\xi}^{T}  \tag{11}\\
\hat{\eta}^{T} \\
\hat{\zeta}^{T}
\end{array}\right]
$$

where $\hat{\xi} \hat{\eta} \hat{\zeta}$ are the versors of the frame $\{\Omega, \xi, \eta, \zeta\}$ axes in the frame $\{\mathrm{O}, \mathrm{x}, \mathrm{y}, \mathrm{z}\}$.
In Fig. 2 the vector $t$ indicates the origin of frame $\mathrm{O}, \mathrm{x}, \mathrm{y}, \mathrm{z}$ in the frame $\Omega, \xi, \eta, \zeta$ and the projection of P on the plane $\xi, \eta$ is represented by point Q , which position vector is $w^{*}$. This last, in homogeneous coordinates is given by:

$$
\widetilde{w}^{*}=\left(\begin{array}{l}
w_{r, \xi}  \tag{12}\\
w_{r, \eta} \\
0 \\
1
\end{array}\right)=\left(\begin{array}{l}
\hat{\xi}^{T} w+t_{\xi} \\
\hat{\eta}^{T} w+t_{\eta} \\
0 \\
1
\end{array}\right)
$$

In the same figure, $n_{r}$ is the versor normal to the image plane $R$, and $n$ will be the same versor in the frame $\{\mathrm{O}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$. The perspective image of vector $w^{*}$ can be obtained by assessing a suitable scale factor. This last depends on the distance $d$ between point $P$ and the image plane, given from the following scalar product:

$$
\begin{equation*}
d=n_{r}^{T} w_{r} \tag{13}
\end{equation*}
$$

Let's indicate with $w_{\{\Omega, \xi, \eta, \zeta\}}$ the vector $w$ in the frame $\{\Omega, \xi, \eta, \zeta\}$ :

$$
\widetilde{w}_{\{\Omega, \xi, \eta, \zeta\}}=\left(\begin{array}{l}
w_{\xi} \\
w_{\eta} \\
w_{\zeta} \\
1
\end{array}\right)
$$

Because $\hat{\xi} \hat{\eta} \hat{\zeta}$ are the versor of the frame $\{\Omega, \xi, \eta, \zeta\}$ axes in the frame $\{\mathrm{O}, \mathrm{x}, \mathrm{y}, \mathrm{z}\}$, it is possible to write the coordinates of the vector $w_{\{\Omega, \xi, \eta, \zeta\}}$ in the frame $\{\Omega, \xi, \eta, \zeta\}$ :

$$
\begin{aligned}
& w_{\xi}=\hat{\xi}^{T} \cdot w=\xi_{x} w_{x}+\xi_{y} w_{y}+\xi_{z} w_{z} ; \\
& w_{\eta}=\hat{\eta}^{T} \cdot w=\eta_{x} w_{x}+\eta_{y} w_{y}+\eta_{z} w_{z} ; \\
& w_{\xi}=\hat{\zeta}^{T} \cdot w=\zeta_{x} w_{x}+\zeta_{y} w_{y}+\zeta_{z} w_{z} .
\end{aligned}
$$

In the frame $\{\Omega, \xi, \eta, \zeta\}$, it's possible to write $w_{r}$ as sum of $w_{\{\Omega, \xi, \eta, \zeta\}}$ and $t$ :

$$
\begin{aligned}
& \widetilde{w}_{r}=\widetilde{w}_{\{\Omega, \xi, \eta, \zeta\}}+\tilde{t}=\left(\begin{array}{l}
w_{\xi}+t_{\xi} \\
w_{\eta}+t_{\eta} \\
w_{\zeta}+t_{\zeta} \\
1
\end{array}\right)= \\
& =\left(\begin{array}{l}
\xi_{x} w_{x}+\xi_{y} w_{y}+\xi_{z} w_{z}+t_{\xi} \\
\eta_{x} w_{x}+\eta_{y} w_{y}+\eta_{z} w_{z}+t_{\eta} \\
\zeta_{x} w_{x}+\zeta_{y} w_{y}+\zeta_{z} w_{z}+t_{\zeta} \\
1
\end{array}\right)
\end{aligned}
$$

an expression of $d$ is:

$$
\begin{align*}
& d=n_{r}{ }^{T} w_{r}=\left(\begin{array}{l}
n_{r, \xi} \\
n_{r, \eta} \\
n_{r, \zeta}
\end{array}\right)^{T} \cdot\left(\begin{array}{l}
w_{\xi}+t_{\xi} \\
w_{\eta}+t_{\eta} \\
w_{\zeta}+t_{\zeta}
\end{array}\right)= \\
& =\left(\begin{array}{l}
n_{r, \xi} \\
n_{r, \eta} \\
n_{r, \zeta}
\end{array}\right)^{T} \cdot\left(\begin{array}{l}
\xi_{x} w_{x}+\xi_{y} w_{y}+\xi_{z} w_{z}+t_{\xi} \\
\eta_{x} w_{x}+\eta_{y} w_{y}+\eta_{z} w_{z}+t_{\eta} \\
\zeta_{x} w_{x}+\zeta_{y} w_{y}+\zeta_{z} w_{z}+t_{\zeta}
\end{array}\right)
\end{align*}
$$

Let's introduce the expressions:

$$
\begin{aligned}
& D_{x}=\frac{\left(\xi_{x} w_{x}+\xi_{y} w_{y}+\xi_{z} w_{z}+t_{\xi}\right) \cdot n_{r, \xi}}{w_{x}} \\
& D_{y}=\frac{\left(\eta_{x} w_{x}+\eta_{y} w_{y}+\eta_{z} w_{z}+t_{\eta}\right) \cdot n_{r, \eta}}{w_{y}} \\
& D_{z}=\frac{\left(\zeta_{x} w_{x}+\zeta_{y} w_{y}+\zeta_{z} w_{z}+t_{\zeta}\right) \cdot n_{r, \zeta}}{w_{z}}
\end{aligned}
$$

it is possible to write:

$$
d=n_{r}^{T} w_{r}=\left(\begin{array}{l}
D_{x}  \tag{13"}\\
D_{y} \\
D_{z} \\
0
\end{array}\right)^{T} \cdot\left(\begin{array}{l}
w_{x} \\
w_{y} \\
w_{z} \\
1
\end{array}\right)=D^{T} \cdot w
$$

In the expression (13") the vector D is:

$$
D=\left(\begin{array}{l}
D_{x} \\
D_{y} \\
D_{z} \\
0
\end{array}\right)
$$

If the image plane R is plane $\xi, \eta$, like in our casẹthen $n_{r}=\left\{\begin{array}{lll}0 & 0 & 1\end{array}\right\}^{T}$ and $D=\{0$ $\left.0 D_{z} 0\right\}$.
As vector $w^{*}$ is given by:
$\widetilde{w}_{p}{ }_{p}=\left(\begin{array}{l}\hat{\xi}^{T} w+t_{\xi} \\ \hat{\eta}^{T} w+t_{\eta} \\ 0 \\ \zeta^{T} w+t_{\zeta}\end{array}\right)$.
The perspective matrix $\left[\mathrm{T}_{\mathrm{p}}\right]$ can be obtained:

$$
\widetilde{w}_{p}^{*}=T_{p} \cdot \widetilde{w} \quad \Rightarrow
$$

$T_{p}=\left[\begin{array}{cccc}\xi_{x} & \xi_{y} & \xi_{z} & t_{\xi} \\ \eta_{x} & \eta_{y} & \eta_{z} & t_{\eta} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & D_{z} & 0\end{array}\right]$
The terms $\mathrm{D}_{\mathrm{x}}, \mathrm{D}_{\mathrm{y}}, \mathrm{D}_{\mathrm{z}}$ assume infinity values if the vector $w$ has one of his coordinates null, but this does not influence on generality of the relation

$$
\widetilde{w}_{p}^{*}=T_{p} \cdot \widetilde{w},
$$

in fact in this case, the term that assume infinity value, is multiplied for zero.
A telecamera can be modelled as a thin lens and an image plane with $C C D$ sensors. The objects located in the Cartesian space emit rays of light that are refracted from the lens on the image plane. Each $C C D$ sensor emits an electric signal, proportional to the intensity of the ray of light on it; the image is made up by a number of pixels, each one of them records the information coming from the sensor corresponding to that pixel.
In order to indicate the position of a point of an image, it's possible to define a frame $u$,v (see Fig. 3) which axes are contained in the image plane. It's possible to associate to each point in the space (which position is given by its Cartesian coordinates), a point in
the image plane (two coordinates) by means of the telecamera. So, the expression "model of the camera" means the transform that associates a point in the Cartesian space to a point in the image space.
In the Cartesian space, a point position is given by three coordinates expressed in length unit while in the image plane the two coordinates are expressed in pixel; this last is the smaller length unit that can be revealed by the camera and isn't a normalized length unit. The model of the camera must take into account this aspect also.
In order to obtain the model of the camera the scheme reported in Fig. 3 can be considered.


Figure 3
Consider a frame xyz in the Cartesian space, the position of a generic point P in the space is given by the vector $w$. Then consider a frame $\xi, \eta, \zeta$ having the origin in the lens centre and the plane $\xi, \eta$ coincident with the plane of the lens; hence, the plane $\xi, \eta$ is parallel to the image plane and $\zeta$ axis is coincident with the optical axis.
Finally consider a frame $u, v$ on the image plane so that $u_{0}$ and $v_{o}$ are the coordinates of the origin of frame $\xi, \eta, \zeta$ expressed in pixel.
The lens makes a perspective transform in which the coordinates of the point in the image plane can be obtained by scaling the coordinates in
the Cartesian space by a factor $-d / f=n_{r}{ }^{T} \cdot w_{r} / f=D^{T} \cdot w / f=D_{z} \cdot w_{z} / f$.
The minus sign is due to the upsetting of the image.
If this transform is applied to vector $w$, a $w_{l}$ vector is obtained:

$$
\begin{equation*}
\widetilde{w}_{l}=T_{p} \cdot F \cdot \widetilde{w} \tag{15}
\end{equation*}
$$

with:
$F=\left[\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{f}\end{array}\right]$
Substantially, the above consists in a changing of the reference frames with matrix $T_{p}$ and a scaling based on the rules of geometric optics previously reported with matrix $F$.
Assumed $x_{1}$ e $y_{1}$ as the first two components of the vector $w_{l}$, the coordinates $u$ and $v$ (expressed in pixel) of $\mathrm{P}^{\prime}$ (image of P ) are:
$\left\{\begin{array}{l}u=\frac{x_{I}}{\delta_{u}}+u_{o} \\ v=\frac{x_{I}}{\delta_{v}}+v_{o}\end{array}\right.$
where $\delta_{u}$ e $\delta_{\mathrm{v}}$ are respectively the horizontal and vertical dimensions of the pixel.
So, by substituting eq. (15) in eq. (17), it comes:
$\left\{\begin{array}{l}u=-\frac{f}{D^{T} w}\left[\left(\frac{1}{\delta_{u}} \cdot \hat{\xi}-\frac{u_{o}}{f} \cdot D\right)^{T} w+\frac{1}{\delta_{u}} \cdot t_{\xi}\right] \\ v=-\frac{f}{D^{T} w}\left[\left(\frac{1}{\delta_{v}} \cdot \hat{\eta}-\frac{v_{o}}{f} \cdot D\right)^{T} w+\frac{1}{\delta_{v}} \cdot t_{\eta}\right]\end{array}\right.$
or:

$$
\left\{\begin{array}{l}
u=-\frac{f}{D_{z} w_{z}} \cdot \frac{w_{r, \xi}}{\delta_{u}}+u_{0} \\
v=-\frac{f}{D_{z} w_{z}} \cdot \frac{w_{r, \eta}}{\delta_{v}}+v_{0}
\end{array}\right.
$$

In homogeneous coordinates, using matrix notation:

$$
\left\{\begin{array}{l}
u  \tag{19}\\
v \\
0 \\
1
\end{array}\right\}=\frac{1}{w_{r, \zeta}}[K] \cdot\left\{\begin{array}{c}
w_{r, \xi} \\
w_{r, \eta} \\
w_{r, \zeta} \\
1
\end{array}\right\}
$$

where matrix K is:

$$
[K]=\left[\begin{array}{cccc}
-\frac{f}{\delta_{u}} & 0 & u_{0} & 0 \\
0 & -\frac{f}{\delta_{v}} & v_{0} & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{array}\right]
$$

Considering equation (10) $\widetilde{\mathrm{w}}_{\mathrm{r}}=\mathrm{T} \cdot \widetilde{\mathrm{w}}$, it is possibile to write (19) in the frame O,x,y,z:
$\left.\left\{\begin{array}{l}u \\ v \\ 0 \\ 1\end{array}\right\}=\frac{1}{D_{z} \cdot w_{z}}[K] \cdot[T]\right\}\left\{\begin{array}{c}w_{x} \\ w_{y} \\ w_{z} \\ 1\end{array}\right\}$
By means of equation (13"), it is possible to write:

$$
D_{z} \cdot w_{z}=\zeta_{x} w_{x}+\zeta_{y} w_{y}+\zeta_{z} w_{z}+t_{\zeta}
$$

If we define the vector $N$, which elements are:

$$
\begin{aligned}
& N_{x}=\zeta_{x} ; N_{y}=\zeta_{y} ; N_{z}=\zeta_{z} ; k=t_{\zeta} \\
& N=\left(\begin{array}{l}
\zeta_{x} \\
\zeta_{y} \\
\zeta_{z} \\
t_{\zeta}
\end{array}\right)
\end{aligned}
$$

(20) becomes:

$$
\left\{\begin{array}{l}
u  \tag{21}\\
v \\
0 \\
1
\end{array}\right\}=\frac{1}{\{N\}^{T} \cdot\{w\}}[K] \cdot[T]\left\{\begin{array}{c}
w_{x} \\
w_{y} \\
w_{z} \\
1
\end{array}\right\}
$$

Equation (21) represents the relation between coordinates ( $u, v$ ) of an assigned point, like a robot endeffector point, expressed in pixels in the image plane, and the coordinates of the same point in the world (Cartesian) frame.

## IV CAMERA MODEL AND D-H ROBOTIC MATRIX

For kinematics' purposes in robotic applications, it is possible to use the Denavit and Hartemberg transformation matrix in homogeneous coordinates in order to characterize the end-effector position in the robot base frame by means of joints variable, this matrix usually contains three zeros and a scale factor in the fourth row. The general expression of the homogenous transformation matrix that allows to transform the coordinates from the frame $i$ to frame $i-1$, is shown in (2):
For a generic robot with $n$ d.o.f., the transformation matrix from endeffector frame to base frame, has the following expression:

$$
T_{n}^{0}=A_{1}^{0} \cdot A_{2}^{1} \cdot A_{3}^{2} \cdot \ldots \ldots \cdot A_{n}^{n-1}
$$

With this matrix it is possible to solve the expression:

$$
\{P\}_{0}=T_{n}^{0} \cdot\{P\}_{n}
$$

where $\{P\}_{0}$ and $\{P\}_{n}$ are the vectors that represent a generic point $P$ in frame 0 and frame $n$.
Can be useful to include this trasformation matrix in camera model (21), in this way it is possible to obtain a perspective representation of the robot in an image plane by means joint coordinates:
$\{u, v\}=\frac{1}{\{N\}_{n}{ }^{T} \cdot\{\widetilde{w}\}_{n}}[K] \cdot\left[T_{n}\right]\{\widetilde{w}\}_{n} \Rightarrow$

$$
\begin{equation*}
\{u, v\}=\frac{1}{\{N\}_{n}{ }^{T} \cdot\left[T_{n}^{0}\right]^{-1}\{\widetilde{w}\}_{0}}[K] \cdot\left[T_{n}\right] \cdot\left[T_{n}^{0}\right]^{-1}\{\widetilde{w}\}_{0} \tag{22}
\end{equation*}
$$

Where:
$\{N\}_{n}$ : vector $\{N\}$ of equation (21) in frame n;
$\left[T_{n}\right.$ ]: matrix [ $T$ ] of equation (10) in frame n ;
Equation (22) represents the relation between end-effector coordinates expressed in pixels, in image plane, and coordinates in the robot joints space. By means this camera model, it is possible to apply the studied algorithm for kinematics calibration.

## Conclusions

The calibration technique can be summarized as follows:
1 Locate a light led on the endeffector and take number $N$ of pictures, each one of them in a different end-effector (hence joint) position;
2 Record, for each position, the values measured from the joint transducers;
3 Consider the generic matrix for a manipulator having that kinematic architecture;
4 Write the direct kinematics equation for each of the picture, considering as unknown the parameters that differ from the joint parameters;
5 Solve the system of equations by a minimum square technique and obtain a number of solutions;
6 Screen the solution by discarding those solution that appear to be incongruent with the given manipulator's structure.
Investigations are still in progress in order to obtain higher precision.

It must be observed that the method has to be linked to a camera calibration method. Early investigations in this sense have been already described in $[5,6,7]$
It must be also observed that from equation (19), from a theoretical point of view, it seems that it could be possible to obtain the mechanical calibration by using just one camera. This last aspect has to be experimentally investigated as, by only one camera, the precision could be very lower.

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