# Kinematics Aspects of Human Body Composite Motion 

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#### Abstract

When the human being is walking/staying or is performing physical exercises on a rolling carpet, the motion of the last one will be transmitted to the body and it will result a composed motion. The influence of such a motion on the human body posture was studied on the joints axes orientation and on the position of the central point of the pelvic girdle (considered as the gravity center of human body). In fact, the gravity center motion is generally considered as a global indicator of the trunk one, because the trunk is rigidly joint to the pelvic girdle. The own motion of the gravity center was obtained as a normal part of the walking process, by modeling the lower limb based on the robotics conventions. But this motion, firstly studied, is modified because of the rolling carpet motion and, as a result, some degenerate postures could appear.


Keywords: lower limb, kinematics model, Denavit-Hartenberg convention, composed motion, hip joint, vibration, transfer matrix.

## 1. Introduction

The lower limb motion was firstly studied by considering a reduced kinematics model, performed by using Denavit-Hartenberg's robotics convention [5]. The joints were simplified considered as follows: hip joint is treated as a superposition of two orthogonal horizontal revolute joints, knee joint is a horizontal revolute one and ankle joint also revolute, having the axis parallel to the precedent one (figure 1 and table 1) [1] [15]. So, the foot length and its own joints were neglected because of small dimensions comparative to femur and tibia lengths, and the reduced kinematics model of the lower limb was obtained [3].


Fig. 1. Reduced kinematics model of the lower limb
The goal of the study was to express the motion amplitude of the point placed on the middle of the pelvic girdle with respect to the global reference frame placed on the rolling carpet. It was also interesting to study the modifications of the axes orientations under the influence of vibrations.
After the usual kinematics modeling was performed, the horizontal motion of the rolling carpet was introduced in the system, the velocity of this last one being 0.5 $m / s$ (figure 2).


Fig. 2 Human body motion along the rolling carpet

## 2. Kinematics modeling

The reduced kinematics model is calculated by using the transfer matrices (1) written for the variables in table 1:

Table 1

| Joint | $\boldsymbol{\theta}_{\boldsymbol{i}}$ | $\boldsymbol{\alpha}_{\boldsymbol{i}}$ | $\boldsymbol{I}_{\boldsymbol{i}}$ | $\boldsymbol{d}_{\boldsymbol{i}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathrm{q}_{1}$ | $0^{\circ}$ | t | L |
| $\mathbf{2}$ | $\mathrm{q}_{2}$ | $0^{\circ}$ | f | 0 |
| $\mathbf{3}$ | $\mathrm{q}_{3}$ | $90^{\circ}$ | 0 | 0 |
| $\mathbf{4}$ | $\mathrm{q}_{4}$ | $90^{\circ}$ | 0 | 0 |
| $\mathbf{5}$ | $180^{\circ}$ | $0^{\circ}$ | $\mathrm{b} / 2$ | 0 |

where the notations are corresponding to those in figure 1 and DenavitHartenberg's convention [10]. Between the last joint (4) and the last reference frame $\left(x_{5} O_{5} y_{5} z_{5}\right)$ there is a constant homogeneous transformation represented by the transfer matrix ${ }^{4} \mathbf{T}_{5}$ [13].

$$
{ }^{\mathbf{0}} \mathbf{T}_{\mathbf{1}}=\left[\begin{array}{cccc}
\cos \theta_{1} & -\sin \theta_{1} & 0 & t \cdot \cos \theta_{1} \\
\sin \theta_{1} & \cos \theta_{1} & 0 & t \cdot \sin \theta_{1} \\
0 & 0 & 1 & L \\
0 & 0 & 0 & 1
\end{array}\right] \quad{ }^{\mathbf{1}} \mathbf{T}_{\mathbf{2}}=\left[\begin{array}{cccc}
\cos \theta_{2} & -\sin \theta_{2} & 0 & f \cdot \cos \theta_{2} \\
\sin \theta_{2} & \cos \theta_{2} & 0 & f \cdot \sin \theta_{2} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$$
\begin{gather*}
{ }^{2} \mathbf{T}_{3}=\left[\begin{array}{cccc}
\cos \theta_{3} & 0 & \sin \theta_{3} & 0 \\
\sin \theta_{3} & 0 & -\cos \theta_{3} & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \quad{ }^{3} \mathbf{T}_{4}=\left[\begin{array}{cccc}
\cos \theta_{4} & 0 & \sin \theta_{4} & 0 \\
\sin \theta_{4} & 0 & -\cos \theta_{4} & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{1}\\
{ }^{4} \mathbf{T}_{5}=\left[\begin{array}{cccc}
-1 & 0 & 0 & -b / 2 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
\end{gather*}
$$

where: $\theta_{i}(i=1, \ldots, 4)$ are joint variables, $f$ is the femur length and $t$ is the tibia length..
The transfer matrices were multiplied, and as a result, the general matrix giving position and orientation of the reference frame $\left(x_{5} O_{5} y_{5} z_{5}\right)$ placed on the middle of the pelvic girdle, was:

$$
\begin{align*}
{ }^{0} \mathbf{G}_{5} & ={ }^{0} \mathbf{T}_{1} \cdot{ }^{1} \mathbf{T}_{2} \cdot{ }^{2} \mathbf{T}_{3} \cdot{ }^{3} \mathbf{T}_{4} \cdot{ }^{4} \mathbf{T}_{5}= \\
& =\left[\begin{array}{cccc}
-c(1+2+3) c 4 & -s(1+2+3) & c(1+2+3) s 4 & -\frac{b}{2} c(1+2+3) c 4+f \cdot c(1+2)+t \cdot c 1 \\
-s(1+2+3) c 4 & c(1+2+3) & s(1+2+3) s 4 & -\frac{b}{2} s(1+2+3) c 4+f \cdot s(1+2)+t \cdot s 1 \\
-s 4 & 0 & -c 4 & -\frac{b}{2} s 4+L \\
0 & 0 & 0 & 1
\end{array}\right] \tag{2}
\end{align*}
$$

where there were noted:

$$
\begin{array}{ll}
\cos \theta_{i}=c i & \sin \theta_{i}=s i \\
\cos \left(\theta_{i}+\theta_{j}\right)=c(i+j) & \sin \left(\theta_{i}+\theta_{j}\right)=s(i+j) \\
\cos \left(\theta_{i}+\theta_{j}+\theta_{k}\right)=c(i+j+k) & \sin \left(\theta_{i}+\theta_{j}+\theta_{k}\right)=s(i+j+k)
\end{array} \quad i, j, k=1, \ldots, 4, ~ l
$$

The kinematics equations describe in fact, the motion of the pelvic girdle with respect to the global reference frame fixed to the rolling carpet. The last one is fixed for instant, in order to have the correct kinematics equations conformably to Denavit-Hartenberg's convention [7].
By taking medium sizes of bones length and imposing the anatomic limits to all angles;

$$
\begin{array}{llll}
t=0.40 m & f=0.50 m & L=0.30 m & b=0.60 m  \tag{3}\\
\theta_{1} & =\left[\begin{array}{lll}
+3 \odot P, & -3 \odot
\end{array}\right] \quad \theta_{2}=\left[\begin{array}{lll}
+9 \oplus, & 0
\end{array}\right] & \theta_{3}=\left[\begin{array}{ll}
+9 \oplus, & -6 \odot
\end{array}\right] \quad \theta_{4}=\left[\begin{array}{lll}
+6 \odot, & -1 \mathscr{O}
\end{array}\right]
\end{array}
$$

the displacement of the origin of $x_{5} O_{5} y_{5} z_{5}$ reference frame was represented with respect to the origin of the global one (figure 3) [11]. The values (3) of angles are describing a complete motion forward-backward of the lower limb [2].
The program calculating the elements of the general matrix ${ }^{0} \mathbf{G}_{5}$ was developed in MatLab 4.2 release, useful to be used because of its efficiency in high order matrices manipulation.
The following lines of program present the manner of the variables and expressions were created:
$\mathrm{t}=0: 0.02: 2 ; \quad$ time interval definition: 101 values from 0 to 2 seconds;
$\mathrm{t} 1=-30: 0.6: 30 ; \quad$ routine to calculate the values interval of angle $\theta_{1}$ from 30 to - 30 degrees,
$\mathrm{t} 3=-60: 1.5: 90 ; \quad \theta_{2}$ from 90 to 0 degrees, $\theta_{3}$ from 90 to -60 degrees and $\theta_{4}$ from 60 to -10 degrees;
$\mathrm{t} 2=0: 0.9: 90$;
$\mathrm{t} 4=-10: 0.7: 60$;
$\mathrm{b}=0.6$;
$\mathrm{L}=0.3$;
$\mathrm{f}=0.5$; given values in meters for tibia and femur length
tib=0.4;
for $\mathrm{i}=1: 101$
$\operatorname{tt} 1(\mathrm{i})=\mathrm{t} 1(102-\mathrm{i})$; calculate values in radians for $\theta_{1}, \theta_{2}, \theta_{3}$ and $\theta_{4}$ variables
$\mathrm{tt} 2(\mathrm{i})=\mathrm{t} 2(102-\mathrm{i})$;
$\mathrm{tt} 3(\mathrm{i})=\mathrm{t} 3(102-\mathrm{i})$;
$\mathrm{tt} 4(\mathrm{i})=\mathrm{t} 4(102-\mathrm{i})$;
$\mathrm{pz4}(\mathrm{i})=\mathrm{L}$;
end
tet $1=\mathrm{tt} 1$ *pi/180;
tet $2=\mathrm{tt} 2 * \mathrm{pi} / 180$;
tet3=tt3*pi/180;
tet $4=\mathrm{tt} 4$ *pi/ 180 ;
px5 $=(-\mathrm{b} / 2) * \cos ($ tet $1+$ tet $2+$ tet 3$) . * \cos ($ tet 4$)+\mathrm{f}^{*} \cos ($ tet $1+$ tet 2$)+$ tib* $\cos ($ tet 1$)$;
py $5=(-\mathrm{b} / 2) * \sin ($ tet $1+$ tet $2+$ tet 3$) . * \cos ($ tet 4$)+\mathrm{f} * \sin ($ tet $1+$ tet 2$)+$ tib* $\sin ($ tet 1$) ;$
$\mathrm{pz5}=(-\mathrm{b} / 2) * \sin ($ tet 4$)+\mathrm{L}$;
pxyz5=sqrt(px5.*px5+py5.*py5+pz5.*pz5);
$\mathrm{nx} 5=-\cos ($ tet $1+$ tet $2+$ tet 3$) . * \cos ($ tet 4$)$;
ny $5=-\sin ($ tet $1+$ tet $2+$ tet 3$) . * \cos ($ tet 4$)$;
$\mathrm{nz} 5=-\sin ($ tet 4$)$;
ox $5=-\sin ($ tet $1+$ tet $2+$ tet 3$)$;
oy $5=-\cos ($ tet $1+$ tet $2+$ tet 3$)$;
ax $5=\cos ($ tet $1+$ tet $2+$ tet 3$) . * \sin ($ tet 4$)$;
ay $5=\sin ($ tet $1+$ tet $2+$ tet 3$) . * \sin ($ tet 4$)$;
az5=-cos(tet4);
Sequence of program to calculate the kinematics equations in the general matrix:

The ".*" operator multiply two arrays element by element since the operator"*" multiply two matrices (arrays) or a matrix (array) by a constant. Each diagram was traced using the PLOT command with specific parameters.

## 3. Normal motion analyze

The human being is walking along the $y_{0}$ axis placed parallel to the length of the rolling carpet, the last one remaining in the same position (null velocity). For each walking step the global duration of 2 seconds was considered [4]. During a step it was also considered that every angle will vary into its own interval (3), by covering it completely [12].


Fig.3. Pelvic girdle displacements conformably to the kinematics model (normal motion)

The curve in figure 3 , a represents the variation with respect to time of the projection of the origin $O_{5}$ on the global reference frame $y_{0}$ axis (line 2 - column 4 of the matrix (2)), given by equation: $p_{y 5}=-\frac{b}{2} c(1+2+3) c 4+f \cdot c(1+2)+t \cdot c 1$. It can be seen in figure 3 , a that the projection $p_{y 5}$ covers the total length of about 0.85 m (between 0.15 m in flexion and 0.7 m in extension) corresponding to a normal walking step for a man.
The relations in column 4 of the matrix (2) are in fact, the parametric equations $p_{x 5}, p_{y 5}, p_{z 5}$ of the origin $O_{5}$, representing the middle of the pelvic girdle during a normal walking on a fixed floor along the $y_{0}$ axis in figure 1. The curve in figure 3, b is calculated as: $p_{5}=\sqrt{p_{x 5}^{2}+p_{y 5}^{2}+p_{z 5}^{2}}$ and represents the implicit shape of the real trajectory of the mentioned point (it means the 3D distance variation between $O_{5}$ and $O_{0}$ ) [14]. It can be seen, that the trajectory is almost a harmonic curve with the reference line at about 0.695 m and the amplitude of 0.075 m . By considering the human being in the vertical static posture where the distance from the pelvic girdle to the floor is: $t+f=0.90 \mathrm{~m}$, it can be seen in the figure $3, \mathrm{~b}$ the global interval of the motion extension. So, during a step with 0.85 m in length,
the value of $p_{5}$ records a maximum variation of 0.25 m (between 0.62 m and 0.87 $m$ ). These values proof the validity of the conceived model, because they are conformably to the reality [6].

## 4. Composed motions

When the rolling carpet starts its own motion with constant velocity of $v=0.5$ $\mathrm{m} / \mathrm{s}$, it is important to study the influence of this translation on the center of gravity motion. In this case, with respect to the global fixed $x_{0} O_{0} y_{0} z_{0}$ reference frame, the new parameters variations are absolute ones. They are obtained by providing the vector sum of the relative and transport laws of motion.
The same parameters represented in figure 3, will remarkably change theirs variations. So, the projection $p_{y 5}$ has the variation in the figure $4, \mathrm{a}$ when human being and rolling carpet advance, and the variation in the figure $4, b$ when the human being advance on a rolling carpet in opposition.


Fig.4. Influence of the rolling carpet translation motion on the projection $p_{y 5}(t)$
As it can be seen in the figure 4 (by comparing with figure 3 ,a) the composed projection $p_{y 5}$ has not only different curves shapes, but even different values. So, in figure $4, \mathrm{a}$ the displacement during flexion becomes 1.17 m for a step, while the displacement during extension keeps the same value of 0.7 m . It means that, during the step duration, the gravity center is moving with respect to the fixed reference frame by covering a distance of 1.87 m . In figure 4,b the composed motion along the rolling carpet length has and aspect forward-backward. The figure shows that, with respect to the fixed reference frame, in the first interval of $0.3 s$ the displacement will be forwarded (limb motion is dominant), in the next interval from $t=0.3 \mathrm{~s}$ to $t=0.9 \mathrm{~s}$ the displacement will be back warded (carpet motion is dominant) and from $t=0.9 \mathrm{~s}$ to $t=1.9 \mathrm{~s}$ the displacement will be also forwarded. This progress will be followed by small recoil during the last 0.1 s . For the assembly of the trajectory the variation is also modified with respect to the situation in figure 3,b.


Fig.5. Influence of the rolling carpet translation motion on the trajectory $p_{5}(t)$
During a step, the extensions between minimum and maximum 3D distance values to the origin of the global reference frame are:

- 0.77 m (between 0.65 m and 1.42 m ) when human being and rolling carpet advance;
- 0.45 m (between 0.75 m and 1.2 m ) when the human being advance on a rolling carpet in opposition.
The influence of the motions composition can also be illustrated for the coxofemoral joint, which normal total displacements $p_{4}(t)$ are presented in figure 6 , and the composed ones in figure 7 , with respect to the same fixed $x_{0} O_{0} y_{0} z_{0}$ reference frame [8] [9].


Fig. 6 Coxofemoral joint displacements conformably to the kinematics model (normal motion)

Step duration [s]

Step duration [s]

Fig. 7 Coxofemoral joint composed displacements
As it can be seen in figure 6, during a step, the value of $p_{4}$ records a maximum variation of 0.23 m (between 0.72 m and 0.95 m ). It is interesting to observe that the distance between the fixed point $O_{0}$ and the origin of the 4-th reference frame in figure 1 , can be greater than $p_{5}$. The difference is introduced by the negative terms of the elements belonging to the 4 -th column of the general matrix ${ }^{0} \mathbf{G}_{5}$ [13]. Taking into account the rolling carpet motion, the absolute displacements in the two different situations are:

- 0.97 m (between 0.65 m and 1.62 m ) when human being and rolling carpet advance;
- 0.75 m (between 0.7 m and 1.2 m ) when the human being advance on a rolling carpet in opposition.
It is also important to remark that the curves shapes are also modified.


## Conclusions

Normal human locomotion can be studied by using robotics conventions, which are able to establish the kinematics equations of the pelvic girdle with respect to any fixed reference frame. The kinematics equations are established taking into account the angular constraints of the joints. The kinematics model could be modified if a greater number of joints are considered, offering to the model a greater number of degrees of freedom. But, the adopted model with 4 degrees of freedom/limb is enough correct to analyze the normal motion as well as, the composed one. It is obvious that any external motion introduced in the mechanical system represented by human lower limb, will affect the normal step geometric configuration. Even the simplest motion, the translation one, could radically transform the distances and theirs variations with respect to time.

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