The Comparative Study of the Cross Displacement Fields for a Kinematical Element Thin Plane Plate with Linear-Elastic Behavior, respectively, with Linear-Viscoelastic Behavior

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Abstract: At the beginning, the paper presents the mathematical models of the cross vibrations for a kinematical element, thin plane plate with linear elastic behavior, respectively, with linear viscoelastic behavior, having a rototranslation motion, member of a kinematical chain. Applying the Laplace integral transform and the double finite in sine Fourier transform, corresponding to the simple bearing, the cross displacement field is determined. This way proceeding, the mechanical and the mathematical model obtained can be assimilated in case of the vibrations for a carriage body of a motor vehicles. In a numerical concrete case, the variation diagrams are presented. Finally, the results obtained for the cross displacements, corresponding to the two behaviors, are compared.

Keywords: vibrations, thin plane plate, linear-elastic behavior, linear-viscoelastic behavior

1 The Motion Mathematical Model

By using the Hamilton's variation principle, in [2] the mathematical model of the cross vibrations in case of a linear-elastic thin plane plate, having a rototranslation motion, is obtained. In Fig. 1, a linear-elastic thin plane plate is represented. Assuming that the plate is in rototranslation motion with a constant angular velocity and submits to the free cross vibrations along this axis, the mathematical model acquires the form:

$$\frac{\partial^4 w}{\partial x_1^4} + 2\frac{\partial^4 w}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 w}{\partial x_2^4} + c^2 \frac{\partial^2 w}{\partial t^2} - c^2 \omega^2 w + c^2 a_0 = 0\,, \tag{1}$$

where:

$\vec{w} = w(x_1, x_2, t) \cdot \vec{i}_3$ is the cross linear elastic displacement;

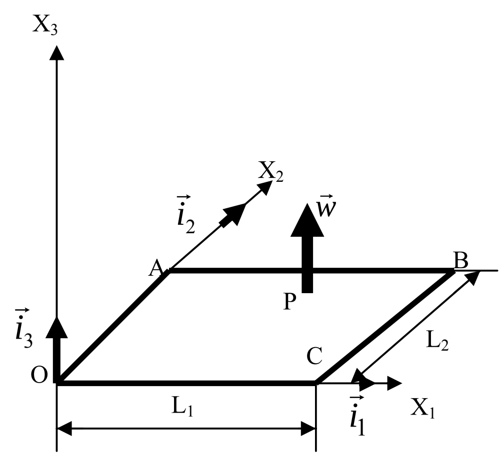

Figure 1
The linear-elastic thin plane plate

$\vec{\omega} = \omega \cdot i_3$ and $\vec{a}_0 = a_{03} \cdot \vec{i}_3$ are the rototranslation elements;

$$c^2 = \frac{12 \cdot \rho \cdot \left(1 - \nu^2\right)}{E \cdot h^2};$$

ρ is the surface specific mass;

ν is the Poisson's coefficient;

E is the Young's modulus;

H is the thickness of the plate.

Let's consider the initial conditions, under their most general form:

$$w(x_1, x_2, 0) = f(x_1, x_2); \; \frac{\partial w}{\partial t}(x_1, x_2, 0) = g(x_1, x_2). \tag{2}$$

In case of linear-viscoelastic behavior of the plate, applying to the equation (1) the Laplace transform in relation with time and considering the rototranslation elements being constant, it results the equation:

$$\frac{\partial^4 \tilde{w}(x_1, x_2, s)}{\partial x_1^4} + 2\frac{\partial^4 \tilde{w}(x_1, x_2, s)}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 \tilde{w}(x_1, x_2, s)}{\partial x_2^4} + c^2 \cdot$$
$$\left[s^2 \tilde{w}(x_1, x_2, s) - s \cdot f(x_1, x_2) - g(x_1, x_2)\right] - c^2 \omega^2 \tilde{w}(x_1, x_2, s) + c^2 a_{03} \frac{1}{s} = 0 \tag{3}$$

where E is substituted with Ẽ(s) (based on the Alfrey and Lee's analogy) witch, for the Maxwell types models, is given by the relation:

$$\widetilde{E}(s) = \frac{a_0 s}{b_0 s + b_1}\,, \tag{4}$$

The equation this way raised represents the mathematical model of the vibrations for the linear-viscoelastic plate.

2 The Dynamical Response

Applying to the mathematical model, obtained based on the Alfrey and Lee's analogy, the double finite Fourier transform in sine, corresponding to the simple bearing on boundary, where the displacements and the bending moments are zero, it results an algebraically equation with the unknown $\widetilde{w}_s^{**}(n,m,s)$ under the form:

$$\begin{aligned}
&\alpha_n^4 \widetilde{w}^{**}(n,m,s) + 2\alpha_n^2 \beta_m^2 \widetilde{w}^{**}(n,m,s) + \beta_m^4 \widetilde{w}^{**}(n,m,s) + \frac{k(b_0 s + b_1)}{a_0 s}\cdot \\
&\left[s^2 \widetilde{w}^{**}(n,m,s) - s f^{**}(n,m) - g^{**}(n,m)\right] - \frac{k(b_0 s + b_1)}{a_0 s}\omega^2 \widetilde{w}^{**}(n,m,s) + \\
&+ \frac{k(b_0 s + b_1)}{a_0 s}\frac{1}{s} a_{03}\, \frac{L_1 L_2 [1-\cos(n\pi)][1-\cos(m\pi)]}{nm\pi^2} = 0.
\end{aligned} \tag{5}$$

Elementary solving the equation (5) and inverting the double finite Fourier transform in sine, it results the solution in Laplace images of the partial differential equation (1):

$$\begin{aligned}
\widetilde{w}(x_1, x_2, s) = {} & \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{nm} \frac{a_{1,nm} s^3 + a_{2,nm} s^2 + a_{3,nm} s + a_{4,nm}}{b_{1,1} s^4 + b_2 s^3 + b_{3,nm} s^2 + b_4 s}\cdot \\
& \cdot \sin(\alpha_n x_1) \sin(\beta_m x_2)
\end{aligned}\,, \tag{6}$$

where:

$a_{1,nm} = \pi^2 \cdot n \cdot m \cdot k \cdot b_0 \cdot f^{**}(n,m);$

$a_{2,nm} = \pi^2 \cdot n \cdot m \cdot k \cdot \left[b_1 f^{**}(n,m) - b_0 g^{**}(n,m)\right]$

$a_{3,nm} = -k\left\{\pi^2 \cdot n \cdot m \cdot b_1 \cdot g^{**}(n,m) + b_0 \cdot L_1 \cdot L_2 \cdot a_{03} \cdot [1-\cos(n\pi)][1-\cos(m\pi)]\right\};$

$a_{4,nm} = -k \cdot b_1 \cdot L_1 \cdot L_2 \cdot a_{03} \cdot [1-\cos(n\pi)][1-\cos(m\pi)]$; $b_{1,1} = k \cdot b_0; b_0 = 3K + G$;

$$\alpha_n = \frac{n \cdot \pi}{L_1}; \quad \beta_m = \frac{m \cdot \pi}{L_2}; \; k = \frac{12 \cdot \rho \cdot \left(1 - \nu^2\right)}{h^2}; \; b_4 = -k\omega^2 b_1 ;$$

$$f^{**}(n,m) = \int_0^{L_1} \int_0^{L_2} f(x_1, x_2) \sin(\alpha_n x_1) \, \sin(\beta_m x_2) dx_1 dx_2 ;$$

$$g^{**}(n,m) = \int_0^{L_1} \int_0^{L_2} g(x_1, x_2) \sin(\alpha_n x_1) \, \sin(\beta_m x_2) dx_1 dx_2 ;$$

$$b_2 = k \cdot b_1; \quad b_1 = \frac{3 \cdot K \cdot G}{\eta}; \; b_{3,nm} = \left(\alpha_n^2 + \beta_m^2\right)^2 \cdot a_0 - k \cdot \omega^2 \cdot b_0; a_0 = 9 \cdot G \cdot K.$$

Further on, are considered many particular cases in which are evidenced the influence of the kinematical parameters on the vibration modes.

a. $\omega = 0$; $f^{**}(n,m) \neq 0$; $g^{**}(n,m) \neq 0$

The solution (6) becomes:

$$\widetilde{w}(x_1, x_2, s) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{nm} \cdot \frac{a_{1,nm} s^3 + a_{2,nm} s^2 + a_{3,nm} s + a_{4,nm}}{b_{1,1} s^4 + b_2 s^3 + b_{3,nm}^1 s^2} \cdot \\ \cdot \sin(\alpha_n x_1) \sin(\beta_m x_2) \quad , \tag{6'}$$

where: $b_{3,nm}^1 = \left(\alpha_n^2 + \beta_m^2\right)^2 \cdot a_0$.

b. $\omega = 0$; $f^{**}(n,m) = 0$; $g^{**}(n,m) = 0$

The solution (6) becomes:

$$\widetilde{w}(x_1, x_2, s) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{nm} \cdot \frac{a_{3,nm}^1 s + a_{4,nm}}{b_{1,1} s^4 + b_2 s^3 + b_{3,nm}^1 s^2} \cdot \\ \cdot \sin(\alpha_n x_1) \sin(\beta_m x_2) \quad , \tag{6''}$$

where:

$$a_{3,nm}^1 = -k \cdot b_0 \cdot L_1 \cdot L_2 \cdot a_{03} \cdot [1 - \cos(n\pi)][1 - \cos(m\pi)] .$$

c. $\omega = 0$; $f^{**}(n,m) \neq 0$; $g^{**}(n,m) = 0$

The solution (6) becomes:

$$\widetilde{w}(x_1, x_2, s) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{nm} \cdot \frac{a_{1,nm}s^3 + a_{2,nm}^1 s^2 + a_{3,nm}^1 s + a_{4,nm}}{b_{1,1}s^4 + b_2 s^3 + b_{3,nm}^1 s^2} \cdot , \qquad (6''')$$
$$\cdot \sin(\alpha_n x_1)\sin(\beta_m x_2)$$

where: $a_{2,nm}^1 = \pi^2 \cdot n \cdot m \cdot k \cdot b_1 f^{**}(n,m)$.

d. $\omega = 0$; $f^{**}(n,m) = 0; g^{**}(n,m) \neq 0$

The solution (6) becomes:

$$\widetilde{w}(x_1, x_2, s) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{nm} \cdot \frac{a_{2,nm}^{(1)} s^2 + a_{3,nm}s + a_{4,nm}}{b_{1,1}s^4 + b_2 s^3 + b_{3,nm}^1 s^2} \cdot \qquad (6^{IV})$$
$$\cdot \sin(\alpha_n x_1)\sin(\beta_m x_2)$$

where: $a_{2,nm}^{(1)} = -\pi^2 \cdot n \cdot m \cdot k \cdot b_0 g^{**}(n,m)$.

e. $a_{03} = 0$ **;** $f^{**}(n,m) \neq 0;$ $g^{**}(n,m) \neq 0$

The solution (6) becomes:

$$\widetilde{w}(x_1, x_2, s) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{nm} \cdot \frac{a_{1,nm}s^2 + a_{2,nm}s + a_{3,nm}^{(1)}}{b_{1,1}s^3 + b_2 s^2 + b_{3,nm}s + b_4} \cdot , \qquad (6^{V})$$
$$\cdot \sin(\alpha_n x_1)\sin(\beta_m x_2)$$

where: $a_{3,nm}^{(1)} = -k\pi^2 nmb_1 g^{**}(n,m)$.

f. $a_{03} = 0$ **;** $f^{**}(n,m) \neq 0;$ $g^{**}(n,m) = 0$

The solution (6) becomes:

$$\widetilde{w}(x_1, x_2, s) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{nm} \cdot \frac{a_{1,nm}s^2 + a_{2,nm}^1 s}{b_{1,1}s^3 + b_2 s^2 + b_{3,nm}s + b_4} \cdot , \qquad (6^{VI})$$
$$\cdot \sin(\alpha_n x_1)\sin(\beta_m x_2)$$

g. $a_{03} = 0$ **;** $f^{**}(n,m) = 0;$ $g^{**}(n,m) \neq 0$.

The solution (6) becomes:

$$\widetilde{w}(x_1, x_2, s) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{nm} \cdot \frac{a_{2,nm}^{(1)} s + a_{3,nm}^{(1)}}{b_{1,1} s^3 + b_2 s^2 + b_{3,nm} s + b_4} \cdot \qquad (6^{\mathrm{VII}})$$
$$\cdot \sin(\alpha_n x_1) \sin(\beta_m x_2)$$

Inverting the Laplace transform in (6'), it results the cross displacement field under the series form:

$$w(x_1, x_2, t) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{nm A_{5,nm}} [\cosh(\Omega_{nm} t) - \sinh(\Omega_{nm} t)] \cdot$$
$$\cdot \{[-1 + \cosh(\omega_{nm} t) + \sinh(\omega_{nm} t)] \cdot A_{1,nm} + [-2 A_{3,nm} (\cosh(\Omega_{nm} t) + \sinh(\Omega_{nm} t)) +$$
$$+ A_{4,nm} (1 + \cosh(\omega_{nm} t) + \sinh(\omega_{nm} t))] A_{2,nm}\} \sin(\alpha_n x_1) \cdot \sin(\beta_m x_2)$$

where: (7)

$$\Omega_{nm} = \frac{b_2 + \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1}}{2 \cdot b_{1,1}} ; \omega_{nm} = \frac{\sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1}}{b_{1,1}} ;$$

$$A_{1,nm} = b_2^2 a_{4,nm} b_{1,1} - b_2 a_{3,nm} b_{1,1} b_{3,nm}^1 - 2 a_{4,nm} b_{1,1}^2 b_{3,nm}^1 - b_2 a_{1,nm} \left(b_{3,nm}^1\right)^2 +$$
$$2 a_{2,nm} b_{1,1} \left(b_{3,nm}^1\right)^2 ; \quad A_{2,nm} = \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1} ;$$
$$A_{3,nm} = A_{3,nm}(t) = b_{1,1} \left(b_2 a_{4,nm} - a_{3,nm} b_{3,nm}^1 - t \cdot a_{4,nm} b_{3,nm}^1\right) ;$$

$$A_{4,nm} = b_2 \cdot a_{4,nm} \cdot b_{1,1} - a_{3,nm} \cdot b_{1,1} \cdot b_{3,nm}^1 + a_{1,nm} \cdot \left(b_{3,nm}^1\right)^2 ;$$

$$A_{5,nm} = 2 \cdot b_{1,1} \cdot \left(b_{3,nm}^1\right)^2 \cdot \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1} .$$

Inverting in (6") the Laplace transform, it results the cross displacement field under the series form:

$$w(x_1, x_2, t) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{2 \cdot b_{1,1}}{nm} \cdot \left\{ B_{1,nm} + \frac{B_{2,nm}}{B_{3,nm}} \cdot [\cosh(\Omega_{nm} t) - \sinh(\Omega_{nm} t)] \right.$$
$$\left. + \frac{B_{4,nm}}{B_{5,nm}} \cdot \left[\cosh\left(\Omega_{nm}^1 t\right) - \sinh\left(\Omega_{nm}^1 t\right)\right] \right\} \sin(\alpha_n x_1) \sin(\beta_m x_2)$$

where: (8)

$$B_{1,nm} = B_{1,nm}(t) = -\frac{b_2 a_{4,nm}}{2 \cdot b_{1,1} \cdot \left(b_{3,nm}^1\right)^2} + \frac{a_{3,nm}^1}{2 \cdot b_{1,1} \cdot b_{3,nm}^1} + \frac{t \cdot a_{4,nm}}{2 \cdot b_{1,1} \cdot b_{3,nm}^1} ;$$

$$B_{2,nm} = b_2 \cdot a_{3,nm}^1 - 2 \cdot a_{4,nm} \cdot b_{1,1} + a_{3,nm}^1 \cdot \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1} ;$$

$$B_{3,nm} = \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1} \cdot \left(b_2 + \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1}\right)^2;$$

$$B_{4,nm} = -b_2 \cdot a_{3,nm}^1 + 2 \cdot a_{4,nm} \cdot b_{1,1} + a_{3,nm}^1 \cdot \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1};$$

$$B_{5,nm} = \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1} \cdot \left(b_2 - \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1}\right)^2;$$

$$\Omega_{nm} = \frac{b_2 + \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1}}{2 \cdot b_{1,1}}; \ \Omega_{nm}^1 = \frac{-b_2 + \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1}}{2 \cdot b_{1,1}}.$$

Inverting in (6''') the Laplace transform, it results the cross displacement field under the series form:

$$w(x_1, x_2, t) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{nmA_{5,nm}} [\cosh(\Omega_{nm} t) - \sinh(\Omega_{nm} t)] \cdot$$
$$\{[-1 + \cosh(\omega_{nm} t) + \sinh(\omega_{nm} t)] \cdot A_{1,nm}^1 + [-2A_{3,nm}^1 (\cosh(\Omega_{nm} t) + \sinh(\Omega_{nm} t)) +$$
$$+ A_{4,nm}^1 (1 + \cosh(\omega_{nm} t) + \sinh(\omega_{nm} t))] A_{2,nm}\} \sin(\alpha_n x_1) \cdot \sin(\beta_m x_2)$$

where:

$$\Omega_{nm} = \frac{b_2 + \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1}}{2 \cdot b_{1,1}}; \omega_{nm} = \frac{\sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1}}{b_{1,1}};$$

$$A_{1,nm}^1 = b_2^2 a_{4,nm} b_{1,1} - b_2 a_{3,nm}^1 b_{1,1} b_{3,nm}^1 - 2a_{4,nm} b_{1,1}^2 b_{3,nm}^1 -$$
$$- b_2 a_{1,nm} \left(b_{3,nm}^1\right)^2 + 2a_{2,nm}^1 b_{1,1} \left(b_{3,nm}^1\right)^2; \quad A_{2,nm} = \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1};$$

$$A_{3,nm}^1 = A_{3,nm}^1(t) = b_{1,1}\left(b_2 a_{4,nm} - a_{3,nm}^1 b_{3,nm}^1 - t \cdot a_{4,nm} b_{3,nm}^1\right);$$

$$A_{4,nm}^1 = b_2 \cdot a_{4,nm} \cdot b_{1,1} - a_{3,nm}^1 \cdot b_{1,1} \cdot b_{3,nm}^1 + a_{1,nm} \cdot \left(b_{3,nm}^1\right)^2;$$

$$A_{5,nm} = 2 \cdot b_{1,1} \cdot \left(b_{3,nm}^1\right)^2 \cdot \sqrt{b_2^2 - 4 \cdot b_{1,1} \cdot b_{3,nm}^1}. \quad (9)$$

Inverting in (6^{IV}) the Laplace transform, it results the cross displacement field under the series form:

$$w(x_1, x_{2,} t) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{nm\left(b_{3,nm}^1\right)^2} \left\{ C_{1,nm} + C_{2,nm} e^{\frac{-t \cdot b_2}{2 \cdot b_{1,1}}} \cosh\left(\frac{1}{2}\omega_{nm} t\right) \right.$$

$$\left. + \left[C_{3,nm} e^{\frac{-t \cdot b_2}{2 \cdot b_{1,1}}} \sinh\left(\frac{1}{2}\omega_{nm} t\right) \right] \frac{1}{\omega_{nm} b_{1,1}} \right\} \sin(\alpha_n x_1) \cdot \sin(\beta_m x_2), \quad (10)$$

where:

$$C_{1,nm} = C_{1,nm}(t) = a_{3,nm} b_{3,nm}^1 - a_{4,nm}\left(b_2 - t \cdot b_{3,nm}^1\right);$$

$$C_{2,nm} = b_2 a_{4,nm} - a_{3,nm} b_{3,nm}^1;$$

$$C_{3,nm} = b_2^2 a_{4,nm} - b_{3,nm}^1\left(b_2 a_{3,nm} + 2 \cdot b_{1,1} a_{4,nm}\right) + 2 a_{2,nm}^{(1)}\left(b_{3,nm}^1\right)^2.$$

In case of the linear-elastic plate are applying to the equation (3) the double finite Fourier transform in sine. This way, it results an algebraically equation with the unknown $\widetilde{w}_s^{**}(n,m,s)$. Elementary solving this equation and inverting the Fourier transform, it results the solution of the equation (1), in Laplace images, under the form:

$$\widetilde{w}(x_1, x_2, s) = \frac{4}{L_1 L_2 \pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{nm} \left\{ \pi^2 c^2 nm \cdot s^2 f_s^{**}(n,m) + \pi^2 c^2 nm \cdot \right.$$

$$\left. \cdot s \cdot g_s^{**}(n,m) - c^2 L_1 L_2 a_0 \cdot \left[1 + (-1)^{n+1}\right] \cdot \left[1 + (-1)^{m+1}\right] \right\} \cdot \quad (11)$$

$$\cdot \frac{1}{s\left[c^2 s^2 + \left(\alpha_n^2 + \beta_m^2\right)^2 - c^2 \omega^2\right]} \cdot \sin(\alpha_n x_1) \sin(\beta_m x_2),$$

where $f_s^{**}(n,m)$, $g_s^{**}(n,m)$ are the double finite transforms in sine of the functions f and g. Finally, in (11) inverting the Laplace transform, it results the cross displacement field $w(x_{1,} x_{2,} t)$, which form depends on the nature of the polynomial roots:

$$P(s) = c^2 s^2 + \left(\alpha_n^2 + \beta_m^2\right)^2 - c^2 \omega^2 \quad (12)$$

Let's consider, for example, that P(s) has imaginary associate roots:

$$\omega \in \left(-\frac{\alpha_n^2 + \beta_m^2}{c}, + \frac{\alpha_n^2 + \beta_m^2}{c} \right).$$

By inverting in (11) the Laplace transform, it is obtained the cross displacement function $w(x_1, x_2, t)$. The cross displacement in the point $P(x_1, x_2) \equiv P\left(\frac{L_1}{2}, \frac{L_2}{2}\right)$, maintaining only two first terms from the development in series of this function is:

$$w\left(\frac{L_1}{2}, \frac{L_2}{2}, t\right) = \frac{4}{L_1 L_2 \pi^2}\left\{\frac{4c^2 L_1 L_2 a_0 \cos\left[\frac{1}{c}\sqrt{-c^2\omega^2 + \frac{\pi^4}{L_1^4 L_2^4}\left(L_1^2 + L_2^2\right)^2}\, t\right]}{\left(\frac{\pi^2}{L_1^2} + \frac{\pi^2}{L_2^2}\right)^2 - c^2\omega^2} - \right.$$

$$\left. - \frac{4c^2 L_1 L_2 a_0}{\frac{\pi^4}{L_1^4 L_2^4}\left(L_1^2 + L_2^2\right)^2 - c^2\omega^2}\right\} + \frac{4}{3L_1 L_2 \pi^2}\left\{\frac{-4c^2 L_1 L_2 a_0}{\left(\frac{\pi^2}{L_1^2} + \frac{9\pi^2}{L_2^2}\right)^2 - c^2\omega^2} \cdot \right. \tag{13}$$

$$\left. \cos\left[\frac{1}{c}\sqrt{-c^2\omega^2 + \frac{\pi^4}{L_1^4 L_2^4}\left(9L_1^2 + L_2^2\right)^2}\, t\right] + \frac{4c^2 L_1 L_2 a_0}{\frac{\pi^4}{L_1^4 L_2^4}\left(9L_1^2 + L_2^2\right)^2 - c^2\omega^2}\right\}.$$

3 Numerical Mapping

Let' consider the concrete case with: $G = 0{,}0118979 \cdot 10^{11}\left[N/m^2\right]$; $\omega = 0$;
$K = 0{,}02424455 \cdot 10^{11}\left[N/m^2\right]$; $\eta = 4{,}5 \cdot 10^{13} \cdot 3600\left[Ns/m^2\right]$; $c^2 = 0{,}148$;
$\rho = 1213{,}3\left[Kg/m^3\right]$ $\nu = 0{,}4$; $h = 3 \cdot 10^{-3}[m]$; $L_1 = 2{,}34[m]$; $L_2 = 1{,}315\,[m]$.

With these values, the function (8) has the representations from the Fig. 2, Fig. 3 and Fig. 4 and the function (13) the representation from the Fig. 5.

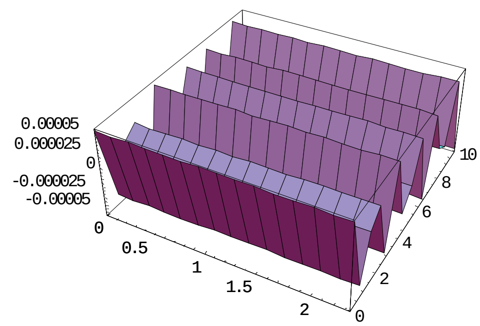

Figure 2
$w = w(x_1, L_2/2, t)$

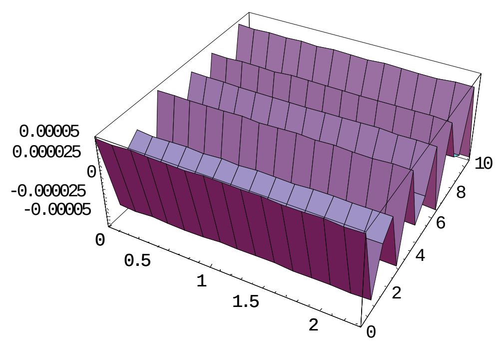

Figure 3
$w = w(L_1/2, x_2, t)$

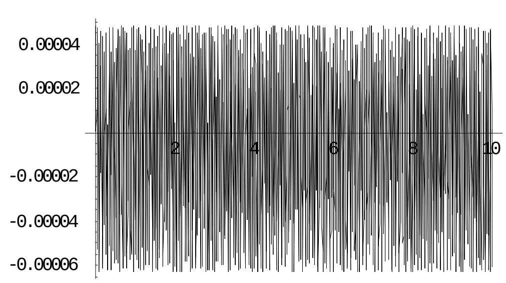

Figure 4
$w = w(L_1/2, L_2/2, t)$

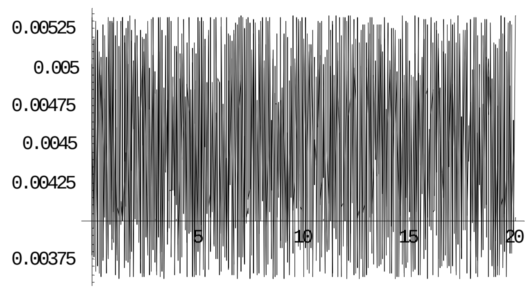

Figure 5
Representation of the function (13): $w = w(L_1/2, L_2/2, t)$

Conclusions

The first four cases have physical significance for particular cases, if are considered especially the vibrations of the carriage body of a motor vehicle, which can be assimilated to the mathematical model presented in this paper. Thus, the present study integrates itself in designing activity of the vehicles, because the determination of the vibration manners conducts to the correct determination of the specific strain tensor and stress tensor components.

References

[1] Băgnaru Dan, Rizescu Sabin, Bolcu Dumitru: Vibratiile sistemelor elastice, Editura Didactică şi Pedagogică, Bucureşti, 1997

[2] Băgnaru Dan, Rinderu Paul:Vibratiile sistemelor mecanice, Ed. Lotus, Craiova, 1998

[3] Băgnaru Dan, Cătăneanu Adina: About the Transversal Displacements Fields of a Thin Linear Elastic Plane Plate, Component Element of a plane Mechanism, in Proceedings of the Eight IFToMM International Symposium on Theory of Machines and Mechanisms, Bucureşti, Romania, August 28 - September 1, 2001, Vol. I, pp. 37-42

[4] Nowacki W.: Dinamica sistemelor elastice, Ed. Tehnică, Bucureşti, 1969

[5] Pierrard, J. M.: Modèles et Fonctions Viscoélastiques Linéaires, La Rhéologie, Paris, 1969

[6] Tschoegl, N.W.: The Phenomenological Theory of Linear Viscoelastic Behavior, Springer-Verlag, Berlin, 1989

[7] Vaicum A.: Studiul reologic al corpurilor solide, Ed. Academiei Române, Bucureşti, 1978