Improved Slope Identification Techniques in Extremum Control

Eva Dulf, Rodica Festila, Clement Festila

Faculty of Automation and Computer Science
Technical University of Cluj-Napoca
Eva.Dulf@aut.utcluj.ro, Rodica.Festila@aut.utcluj.ro,
Clement.Festila@aut.utcluj.ro

Abstract: In the most control problems, the task of the regulator is to keep some variables at constant values. The reference values are easily determined. A number of processes should have as high efficiency as possible and their performance can be improved by adjusting some plant variables as to maximize or to minimize the performance criterion. Such kind of control is called extremum control. The major chacateristic of these systems is a static response curve, relating the output to input with nonlinear character and with an extremum. The objective of the extremum control system is to keep the process on the extremum point, despite changes in the process or influence of disturbances. In order to achieve this aim, the slope seeking (identification) is a major job of the control engineer.

Keywords: nonlinear extremum systems, slope seeking, signal demodulation

1 Introduction

Extremum control system applications are specific to plants with a nonlinear static response curve ("static characteristic") $\varphi=f(\mu)$ with extremum (maximum or minimum). It is obvious, it is an ambiguous function: the same output value $\varphi=\varphi^*$ may be obtained with two (or more) input values: (μ_1, μ_2). The control signal has different direction (different derivative) if $\mu=\mu_1$ or $\mu=\mu_2$. The main problem which must be solved is to "reject" the ambiguity in order to build a correct control signal.

The desired set point is unknown a priory and/or varies with different conditions in a complex way. It is necessary to define an *objective function* of the system and than to maximize or minimize this function, as effect of a proper control signal.

For the most frequent cases, the extremum control system is not the "main aim" of the control application. A different output variable must be stabilized – or has to truck a specific reference – and the extremum control system is attached to the main control system in order to maximize or to minimize the objective function: fuel consumption, electrical energy, etc.

A lot of effort was put into the research concerning extremum controllers in the 1950-1960[1,5 7], but due to the lack of the useful results in analysis (stability, behavior, etc.), the number of application decreased. A series of papers in the period 1990-2000[7] – and specially papers of Krstic [2,3,4,6] – treated problems concerning the stability criteria for a given perturbation scheme. The cheaper and more powerful computing power extended the area of actual application [5,7].

2 Slope Identification in Extremum Control

The nonlinear input-output steady-state dependence with extremum may be modeled by the second degree equation [1,2,7]]:

$$\varphi = \varphi^{*} + E\left(\mu - \mu^{*}\right)^{2} \tag{1}$$

Starting the loop operation from the initial condition (μ_i), the manipulated variable of the extremum control system (μ_m) has to act till:

$$\mu_{st} = \mu_m = \mu^{*}$$

and the objective function (φ) reached the minimum.

The main idea of the slope identification (or, equivalently, to reject the ambiguity) is to apply the “gradient methods”. There are other methods too, but the “gradient methods” are the most frequent, like [1,2,7]]: switching systems, self-driving system and perturbation method.

The authors develop the last method, but the perturbation signal will be called “test signal” and is related to the series of papers of Krstic and co-workers [2,3.4]]. The principle of the extremum control system operation with sinusoidal test signal $A\sin(\omega_0 t)$ and $H_i(s)=1$ is presented in Figure 1.

The frequency (ω_0) is “sufficiently large” [2,6] and not equal to any frequency in noise and not equal to any imaginary axis zeros of $H_i(s)$.

In accord to [2], the high-pass filter $c^{*}_{o}(s)$ transfers only the AC- component from z(t):

$$A \cdot E \cdot \varepsilon_{\mu} \cdot sin\left(\omega_o t - d\right) \tag{2}$$

where (ε_μ) is the “error” $\varepsilon_{\mu} \cong \left(\mu - \mu^{*}\right)$, i.e. the deviation from the point of extremum.

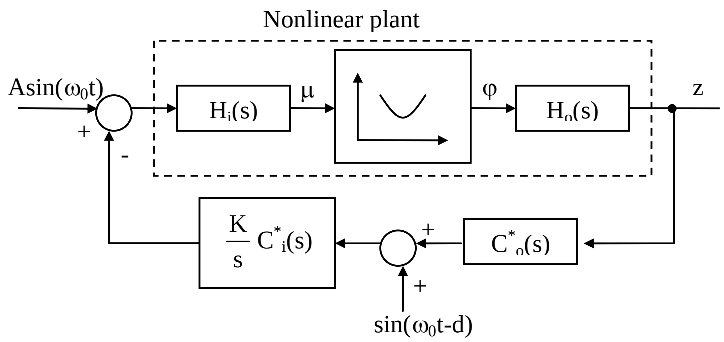


Figure 1
Extremum seeking algorithm with sinusoidal test signal

In order to determine the value of (ε_μ), a demodulation by multiplication was used [2]:

$$A \cdot E \cdot \varepsilon_\mu \cdot sin(\omega_o t - d) \cdot sin(\omega_o t - \beta) = \frac{A \cdot E \cdot \varepsilon_\mu}{2} \cdot [1 - cos\, 2(\omega_o t - d)] \tag{3}$$

with $\beta \cong d$. An integrator $\frac{K}{s} \cdot C_i^*(s)$ (for simplicity with $C_i^*(s) = \frac{1}{Ts+1}$) rejects the AC component with the frequency ($2\omega_o t$). The output signal (μ_m) acts as the manipulated variable and moves the operating point toward the extremum.

The stability conditions for SISO systems given in [2,3] are usually fulfilled [1,5,7] hence the extremum points can be reached.

The "original" scheme given by Krstic [2.3.4] is based on some particularities:

a – the AC component, equation (2), is selected by a wash-out ("derivative" or "high-pass") filter:

$$C_o^*(s) \approx \frac{s}{s+h} \tag{4}$$

b – the deviation (ε_μ) is obtained using a "demodulation by multiplication", equation (3).

It is well-known that the high-pass filter emphasizes the noise, which is always present in industrial application. A low-pass filter before the wash-out filter (4) will alleviate the useful signal. In order to avoid this conflict, the authors propose a structure without derivative element, based on the equation:

$$1 - \frac{1}{Ts+1} = \frac{Ts}{Ts+1} \tag{5}$$

by which a low-pass filter with small time-constant ($\tau<<T$) is now useful:

$$\frac{1}{\tau s+1}-\frac{1}{(Ts+1)(\tau s+1)}=\frac{Ts}{Ts+1}\cdot\frac{1}{\tau s+1} \qquad (6)$$

On the other hand, the "direct demodulation" using switches seems to be a cheaper and simple solution, compared with the solution given in [2,3,6]. Actual analog or hybrid electronic circuits allow an effective solution in order to obtain the variable (ε_μ), figure 2, with $\beta \approx d$.

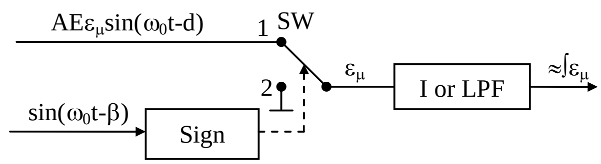


Figure 2
Direct demodulation using switch

If $\varepsilon_\mu(t)>0$ — that is, the operating point is, for instance, in the right-side of the extremum (and the slope $d\varphi/d\mu$ has a certain sign), the ($\int\varepsilon_\mu$) must decrease in order to reach the extremum, being negative.

For $\varepsilon_\mu(t)<0$ — the extremum will be reached if ($\int\varepsilon_\mu$)>0 and the slope($d\varphi/d\mu$) has an opposite sign.

3 The Problem of Phase Displacements (d) and (β)

An input sinusoidal signal $A\sin(\omega_0 t)$ gives an ouput signal with a phase angle displacement:

$$d=\angle H_o(j\omega_0) \qquad (7)$$

usually, negative. If the transfer function $H_o(s)$ is exactly known and $C_o(s)$ is imposed, the phase-angle (d) may be known with a high degree of accuracy. The same phase-angle (β) has to control the "demodulator", equal in version with multiplication or in version with direct demodulation: ($\beta \approx d$). If the parameter or structure of $H_o(s)$ changes, an adaptive extremum control system must be used. The exact value of (d) is very important: an error $\beta^*=\beta\pm\pi=d\pm\pi$ is fatal because the "negative" feedback on the path of the manipulated variable ($\mu_m(t)$) will change in "positive" feedback, and the operating point will move off the extremum (a typical case of instability).

4 Proof by Simulation of the Proposed Solution Advantages

Using SIMULINK, the authors simulate the extremum control systems in two versions: the original scheme given in [2, 4] with derivative filter and multiplier and the modified version without derivative element and with direct demodulator, using switches.

The *common elements* are:

The transfer function $H_o(s)$: $H_o(s) = \frac{2}{(4s+1)(6s+1)}$

The nonlinear (extremum) input output dependences:

$$\varphi_1(\mu) = 2.5 + (\mu - 3)^2 \quad (8)$$

$$\varphi_2(\mu) = 3 + (\mu - 2.5)^2 \quad (9)$$

in order to observe the effects of the variations in main parameters (μ^*) and (φ^*).

The test signal:

$$A \cdot sin(\omega_0 t) = 0.2 \cdot sin(0.5t) \quad (10)$$

A noise signal added to the output:

$$z_\Sigma = z(t) + n(t) \quad (11)$$

where n(t) is a white noise signal, with the power spectral density of 0.1W/Hz.

The simulation strategy is as follows.

- The extremum control system is separated from the main control system.
- An initial value (μ_i) is given: this determines the initial position of the operating point relative to the extremum. Initially, the slope identification “mechanism” is blocked (A=0).
- At the moment t_1=50, the slope identification process is started and the manipulated variable (μ_m) “closes” the operating point to the extremum. The nonlinearity with extremum is given by the equation (9).
- At the moment t_2=200, the slope identification still works, but the nonlinearity is switched to the equation (8).

The schematic diagrams of the extremum control systems are given in Figure 3.

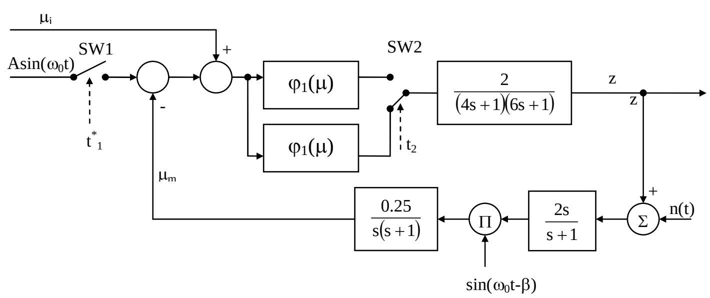
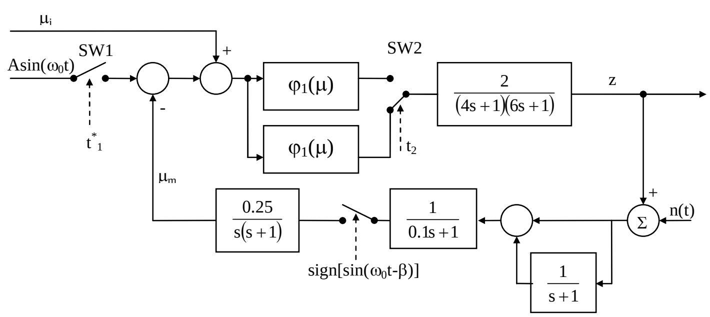


Figure 3

a) “Original” structure from [2] of the Extremum Control System (“multiplier”)



b) Modified structure of the Extremum Control System (“switch”)

The SIMULINK schemes, in accord to the Figures 3a and 3b, are presented in the Figures 4a and 4b.

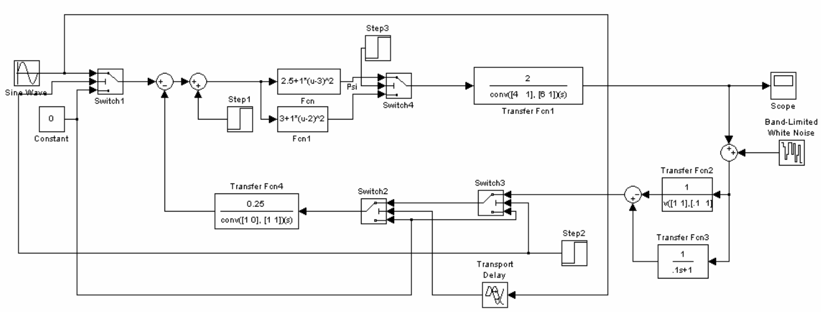


Figure 4a

SIMULINK scheme (Figure 3a)

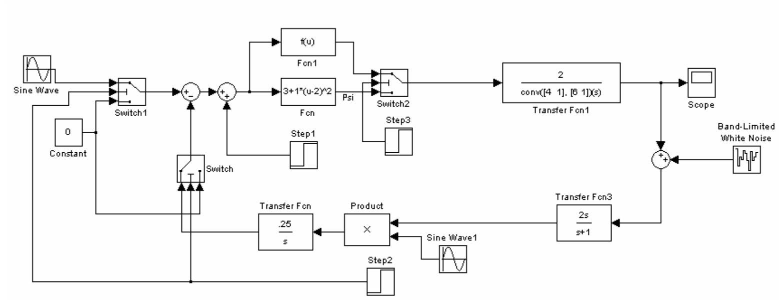


Figure 4b

SIMULINK scheme (Figure 3b)

The results of the simulation are presented in a simplified form in Figures 5 and 6. With the gain of the $H_o(s)$ equal to 2, the extremum point has a value of 5 (gain×φ^*=2×2.5), in accord to the static response curve from equation (8) and a value of 6 (gain×φ^*=2×3) in accord to the static response curve from equation (9). Figure 5 presents the evolutions of the outputs z(t) for the studied cases. Figure 6 presents the effect of the noise as output of the filtered signal z(t).

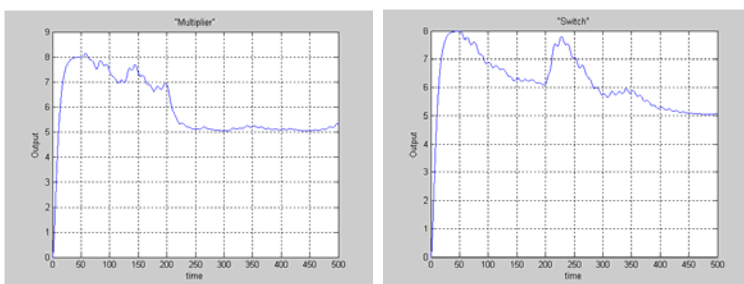


Figure 5

Output z(t) for the original ("multiplier") and modified ("switch") scheme

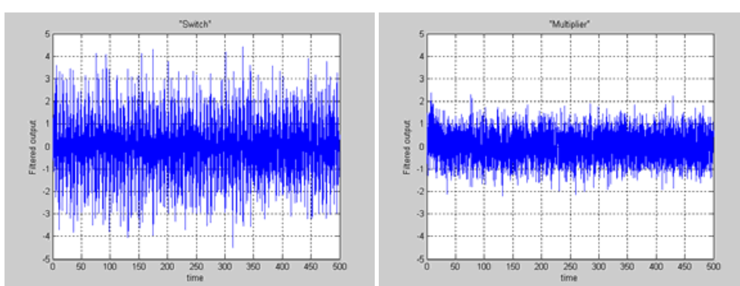


Figure 6

Filtered output for the original ("multiplier") and modified ("switch") scheme

Conclusions

Although the basic of both schemes (original and modified) are in fact the same, some improvements were ascertained. The evolution speed of the operating point to the extremum is *faster* in the case of the modified scheme: see Figure 5. As example, at t=200 when the extremum must have the value of 6, were obtained the values 6.88 for the original scheme and 6.10 for the modified scheme. At the time t=500, the extremum has the value of 5, but the obtained values are: 5.35 for the original scheme and 5.08 for the modified scheme.

What about the noise filtering, the lack of the derivative element in the modified scheme leads to the reduction of the noise level in the AC-component from the output z(t). As it is possible to see from the figure 6, the middle value of the noise is bounded in the domain +5 /-5 in the original case and in the domain +2 /-2 in the modified scheme.

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