Fuzzy Logic Control Problems Simulation Based on Parametrizied Operators

Márta Takács, Sándor Szénási, Ágnes Szeghegyi

Budapest Tech
Bécsi út 96/B H-1034 Budapest, Hungary
takacs.marta@nik.bmf.hu
szenasi.sandor@nik.bmf.hu
szeghegyi.agnes@kgk.bmf.hu

Abstract. Based on the definitions and theorems for lattice ordered monoids and left continuous uninorms and t-norms, certain parameter dependent distance-based operators are focused on, with the help of which the uninorm-residuum based and sinilarity measures based approximate reasoning system becomes possible in Fuzzy Logic Control (FLC) systems. [1]

Keywords: compositional rule of inference, fuzzy rule base system, distance based operators, residual operators

1 Introduction

The real world systems are multi-criterial and multipart and the decision processes become increasingly vague and hard to analyze [4]. The human brain possesses some special characteristics that enable it to learn and reason in a vague and fuzzy environment.In fuzzy control system the system state is described by a fuzzy rule base system [3], and the relationship between fuzzy rule base system, system output and system input is modelled by compositional rule of inference with several system parameters. The concept of approximate reasoning in the known framework of the linguistic information was introduced by Zadeh.

In the fuzzy rule based control theory and usually in the approximate reasoning, as well as in the covering over of fuzzy rule base input and rule premise of a rule determine the importance of that fuzzy rule and the rule output, too. The practical realization of that notion usually depends on the application. The Mamdani type controller is based on Generalized Modus Ponens (GMP) inference rule, and the

[1] This work was supported by the Hungarian Research Foundation, OTKA T 048756.

rule output is given with a fuzzy set, which is derived from rule consequence, as a firing level dependent cut of them. Engineering applications are satisfied with the minimum operator, but from a mathematical point of view it is interesting to study the behavior of other t-norms in inference mechanism. Uninorms continue to bring new possibilities in fuzzy systems models [5]. The modified Mamdani's approach, with similarity measures between rule premises and rule input, does not rely on the compositional rule inference any more, but still satisfies the basic conditions supposed for the approximate reasoning for a fuzzy rule base system. From mathematical point of wiev, and having results from [7], we can introduce residuum-based and sismlarity measures based inference mechanism using distance-based uninorms [8].

2 Modified Distance-based Operators

The distance-based operators, which dwpwnds on the parameter *e*, can be expressed by means of the min and max operators as follows (the only modificaton on the original published distance based operators in [6] is the boundary condition for neutral element *e*):

th*e maximum distance minimum operator with respect* to $e \in]0,1]$ is defined as

$$max_e^{min} = \begin{cases} max(x,y), & \text{if } y > 2e - x \\ min(x,y), & \text{if } y < 2e - x \\ min(x,y), & \text{if } y = 2e\text{-}x \end{cases}$$

the *minimum distance minimum operator* with respect to $e \in [0,1[$ is defined as

$$min_e^{min} = \begin{cases} min(x,y), & \text{if } y > 2e - x \\ max(x,y), & \text{if } y < 2e - x \\ min(x,y), & \text{if } y = 2e\text{-}x \end{cases}$$

the *maximum distance maximum operator with respect to* $e \in]0,1]$ is defined as

$$max_e^{max} = \begin{cases} max(x,y), & \text{if } y > 2e - x \\ min(x,y), & \text{if } y < 2e - x \\ max(x,y), & \text{if } y = 2e\text{-}x \end{cases}$$

the *minimum distance maximum operator with respect to* $e \in [0,1[$ is defined as

$$min_e^{max} = \begin{cases} min(x,y), & \text{if } y > 2e - x \\ max(x,y), & \text{if } y < 2e - x \\ max(x,y), & \text{if } y = 2e\text{-}x \end{cases}.$$

The distance-based operators have the following properties

max_e^{min} *and* max_e^{max} are uninorms,

the dual operator of the uninorm max_e^{min} *is* max_{1-e}^{max} *, and*

the dual operator of the uninorm max_e^{max} *is* max_{1-e}^{min}.

A binary operator V is called idempotent, if $V(x,x) = x, (\forall x \in X)$. It is well known, that the only idempotent t-norm is *min*, and the inly t-conorm is *max*.

In [7] has studied two important classes of uninorms: the class of left-continuos and the class of right-continuos ones.

If we suppose a unary operator *g* on set [0,1], then *g* is called

(i) sub-involutive if $g(g(x)) \leq x$ for $(\forall x \in [0,1])$, and

(ii) super-involutive if $g(g(x)) \geq x$ for $(\forall x \in [0,1])$.

A binary operator U is a conjunctive left-continuous idempotent uninorm with neutral element $e \in]0,1]$ if and only if there exist a super-involutive decresing unary operator *g* with fixpoint *e* and $g(0) = 1$ such that U for any $\forall (x,y) \in [0,1]^2$ is given by

$$U(x,y) = \begin{cases} min(x,y) & if \quad y \leq g(x) \\ max(x,y) & elsewhere \end{cases}.$$

A binary operator U is a dijunctive right-continuous idempotent uninorm with neutral element $e \in [0,1[$ if and only if there exist a sub-involutive decresing unary operator *g* with fixpoint *e* and $g(1) = 0$ such that U for any $\forall (x,y) \in [0,1]^2$ is given by

$$U(x,y) = \begin{cases} max(x,y) & if \quad y \geq g(x) \\ min(x,y) & elsewhere \end{cases}$$

Operator $max_{0.5}^{min}$ is a conjunctive left-continuous idempotent uninorm with neutral element $e \in]0,1]$ with the super-involutive decresing unary operator $g(x) = 2e - x = 2 \cdot 0.5 - x \Rightarrow g(x) = 1 - x$.

Operator $min_{0.5}^{max}$ is a disjunctive right-continuous idempotent uninorm with neutral element $e \in \left]0,1\right]$ with the sub-involutive decresing unary operator $g(x) = 2e - x = 2 \cdot 0.5 - x \Rightarrow g(x) = 1 - x$.

3 Residual Implicators for Uninorms

Residual operator R_U, considering the uninorm U, can be represented in the following form:

$$R_U(x,y) = sup\{z \mid z \in [0,1] \wedge U(x,z) \leq y\}.$$

Uninorms with neutral elements $e = 0$ and $e = 1$ are t-norms and t-conorms, respectivly, and related residual operators are weidly discussed. In [7] we also find suitable definitions for uninorms with neutral elements $e \in \left]0,1\right[$.

If we consider a uninorm U with neutral element $e \in \left]0,1\right[$, then the binary operator R_U is an implicator if and only if $(\forall z \in \left]e,1\right[)(U(0,z) = 0)$. Furthermore R_U is an implicator if U is a disjunctive right-continuous idempotent uninorm with unary operator g satisfying $(\forall z \in [0,1])(g(z) = 0 \Leftrightarrow z = 1)$.

The residual implicator R_U of uninorm U can be denoted by Imp_U.

3.1 Residual Implicators of Distance-based Operators

Consider the conjunctive left-continuous idempotent uninorm $max_{0.5}^{min}$ with the unary operator $g(x) = 1 - x$, then its residual implicator $Imp_{max_{0.5}^{min}}$ is given by

$$Imp_{max_{0.5}^{min}} = \begin{cases} max(1-x, y) & if \quad x \leq y \\ min(1-x, y) & elsewhere \end{cases}.$$

4 Degree of Coincidence in Inference Mechanism

In system control intuitively one would expect: let's make the powerful coincidence between fuzzy sets stronger, and the weak coincidence even weaker. The distance-based operators group satisfy that properties, but the covering over $A(x)$ and $A'(x)$ are not really reflect by the *sup* of the membership function of the $T_e^{max}(A(x),A'(x))$.

Hence, and because of the non-continuos property of distance-based operators, it was unreasonable to use the classical degree of firing, to give expression to the coincidence of the rule premise (fuzzy set *A*), and system input (fuzzy set *A'*), therefore a Degree of Coincidence (*Doc*) for those fuzzy sets has been initiated. It is nothing else, but the proportion of area under membership function of the distance-based intersection of those fuzzy sets, and the area under membership function of the their union (using *max* as the fuzzy union).

$$Doc = \frac{\int_X T_e(A, A')dx}{\int_X \max(A, A')dx}$$

This defnition has two advantages:

It consider the width of coincidence of *A* and *A'*, and not only the "height", the *sup*, and the rule output is weighted with a measure of coincidence of *A* and *A'* in each rule.

The rule output fuzzy set *B'* is achieved as a cut of rule consequence *B* with *Doc*.

$$B'(y)=T_e^{min}(B(y), Doc) \quad \text{or}$$

$$B'(y)=T_e^{max}(B(y), Doc)$$

It is easy to prove that $Doc \in [0,1]$, and $Doc=1$ if *A* and *A'* cover each other, and then $B'(y)=B(y)$, and $Doc=0$ if *A* and *A'* have no point of contact, and then $B'(y)=0$.

The FLC rule base output is constructed as above explained. The output is constructed as a crisp value calculated from rule base output, which is an aggregation of all rule consequences $B_i'(y)$ in rule base. For aggregation, distance based operators S_e^{min} or S_e^{max} can be used.

An additional possibility is if the cut $B'(y)$ of the rule concequence $B_i(y)$ is calculated from the

$$Doc = \frac{\int_Y B'(y)dy}{\int_Y B(y)dy}$$

expression.

Based on this, for triangular membership functions $A(x)$, $A(x)$, $B(y)$, we have

$$B'(y) = \max\left(B(y), 1 - \sqrt{1 - Doc}\right).$$

$B'(y)$ is obtained as a weighted fuzzy set, and the weight parameter *Doc* depends on $\int T_e^{max}(A(x),A'(x))dx$. It is a measure number related to the area under membership function $T_e^{max}(A(x),A'(x))$. Taking this fact into consideration, a connection between *Doc* type of inference mechanism and generalized fuzzy measures and integrals is being researched [1], [2].

5 The Simulation System

The simulation system will be presented, in which system behavior is analyised depending on change in FLC by parameter choosing, using distance based operators.

The system was built in MATLAB-SIMULINK environment, and the programs, that substitute the FLC-s, were written in MATLAB. The SOURS is a step function (*Step Input*), with the time step 0.1 sec, with the starting value 0, and the final value 1.*e* and *y* were transformed into the interval [-1,1]. The system to be controlled is modeled by a first order differential equation, $q' = k_1\left(y + k_2 q\right)$. During the simulation all of the components of the system were fixed except the FLC. The goal is, achieving the desired (input) step function as fast is it possible. The questions are: does this function reaches 1, as well as how stable this system is.

5.1 The FLC Model

In the theory of approximate reasoning the knowledge of system behavior and system control can be stated in the form of if-then rules: if x is A_i then y is B_i

Usually the general rule consequence for one rule from the *i*-th rule base system is obtained by

$$Bi'(y)=\sup_{x\in X}(OPDis2(A'(x),OPDis2(Ai(x),Bi(y))).$$

The connection *OPDis1* and *OPDis2* are generally defined, and they can be some type of fuzzy disjunctive operators.

The Mamdani inference rule states that the membership function of the consequence in the *i*-th rule B_i' is defined by

$$Bi'(y)=\sup_{x\in X}(OPDis\ (A'(x),\ OPDis\ (Ai(x),Bi(y)))$$

where *OPDis* is a fuzzy disjunctive operator.

Using the operator properties, from the above expression follows

$$Bi'(y)=OPDis\ (\sup_{x\in X}(OPDis\ (A'(x),Ai(x))),Bi(y)).$$

Generally speaking, the consequence (rule output) is given with a fuzzy set $B'(y)$, which is derived from rule consequence $B(y)$, as a cut of the $B(y)$. This cut can be

- the generalized degree of firing level of the rule:

  $$DOF=\sup_{x\in X}(OPDis\ (A'(x),A(x))),$$

  considering actual rule base input $A'(x)$, and usually depends on the covering over $A(x)$ and $A'(x)$. But first of all it depends on the *sup* of the membership function of $OPDis\ (A'(x),A(x))$, or

- degree of conicidence, defined in the section 4.

Rule base output B'_{out} is an aggregation of all rule consequences $B_i'(y)$ from the rule base. As aggregation operator a conjunctive fuzzy operator is usually used.

$$B'_{out}(y)=$$
$$OPCon(B_n'(y),OPCon(B_{n-1}'(y),OPCon(....,OPConS(B_2'(y),B_1'(y))))).$$

The crisp FLC output y_{out} is constructed as a crisp value calculated with a defuzzification method, from rule base output, for example with the center of gravity method, given by

$$y_{out}=\frac{\int_Y B'_{out}(y)\cdot y dy}{\int_Y B'_{out}(y)\cdot dy}.$$

It can be conclude, that in approximate reasoning the (*OPDis, OPCon*) pair of operators are used.

5.2 The Investigated System

The rule base of the FLC, analysed in this case contains 5 rules, well-known from fuzzy applications. The operators OPDis and OPCon are chosen from the group of distance based operators.

The maximum distance minimum operators are from disjunctive operators group, the minimum distance maximum operators are from conjunctive operators group.

Considering the structure of distance based operators, namely that they are constructed by the min and max; it was worth trying to move away from the strictly applied max (disjunctive) and min (conjunctive) operators pair in approximate reasoning. Therefore, in the simulation system different operators from the group of distance based operators were applied as disjunctive and conjunctive. Moreover, the distance based operators are parametrized by the parameter *e*, therefore the program (*S* -function), which performs the task of FLC in the simulation system, has global, optional, variables (*OPDis, OPCon*, *e*), where $Opdis$ is the operator applied by GMP, and the $OPCon$ is the aggregation operator for the calculation of the B'_{out}. The neutral element of the *OPDis* operator is parameter *e*, and the neutral element of the *OPCon* operator is parameter 1-*e*.

5.3 The System Behavior with Several (OPDis, OPCon, e) Triples in FLC

The simulation system was built in MATLAB-SIMULINK environment, and the program (S-function) of the FLC model in C programming language. This program runs over in every simulation step, get one crisp number from the simulation system and gives back a crisp number to the system. The possibilities of the programm are:

- choosing of the OPDis and OPCon operators from the group of the distance based operators,
- sliding of the parameter *e* of the distance based operators (in the interval [0,1]),
- sliding of the center of the fuzzy sets of rule premises and rule consequences in rules of the fuzzy rule base. The rule bases contains 5 rules.

5.4 The Simulation Results

The criteria for comparison of the simulation results are the following:

- How fast does it reach the intensity of 1?
- How precisely does it reach the intensity 1?
- How significant is the dispersion around intensity 1?
- Does the irregular behaviour repeat periodically?

The significanrt results are:

If

$$(OPDis, OPCon, e) = \left(\max_e^{\min}, \min_{1-e}^{\max}, e\right),\ (e \neq 0{,}5),\ \text{(case (M2.8.)) or}$$

$$(OPDis, OPCon, e) = \left(\max_e^{\min}, \min_{1-e}^{\min}, e\right),$$

case (M2.5.), the conclusions are:

the step function does not achieve the intensity 1, but the system is stable. The time from the start to stability is cca 0.8 seconds.

If

$$\left(OPDis, OPCon, e\right) = \left(\min_{e}^{\max}, \min_{1-e}^{\min}, e\right),\ \left(e \neq 0{,}5\right), \text{(case (M2.9.)) or}$$

$$\left(OPDis, OPCon, e\right) = \left(\min_{e}^{\min}, \min_{1-e}^{\min}, e\right),$$

case (M2.13.), the conclusions are:

the step function from FLC output fails to reach intensity 1 again (it is cca. 0,5), yet the system is stable. The time from the start to stability is cca 0.2 seconds, the simulation process is fast.

If

$$\left(OPDis, OPCon, e\right) = \left(\max_{e}^{\max}, \min_{1-e}^{\max}, e\right),\ \left(e \neq 0{,}5\right),$$

(case (M2.3.)), the conclusions are:

the step function from FLC output reaches the intensity 1, and the system portrays urregularities periodically.

If

$$\left(OPDis, OPCon, e\right) = \left(\max_{e}^{\min}, \max_{1-e}^{\min}, e\right),$$

$\left(e = 0{,}5\right)$, (case (M2.6.)), the conclusions are:

the step function from FLC output reaches the intensity 1, and the system stay stable after 0,5 seconds. This choice of the parameter and operators is the best for the investigated system.

The output is in several cases, but does not have the sufficient intensity, at other times, it does have the sufficient intensity, but has a periodical irregularity. From this it can be concluded that the other elements (gains, coeficients) of the simulation system (and of the real system) can be changed to achieve the desired state in a short period of time.

Conclusions

Despite the fact, that Mamdani's approach is not entirely based on compositional rule of inference, it is nevertheless very effective in fuzzy approximate reasoning. Because of this it is poosible to apply several t-norms, or, as in this case, uninorms and distance based operators. This leads to further tasks and problems, because in any case there must be a system of conditions that is to be satisfied by the new model of inference mechanism in fuzzy systems.

References

[1] B. De Baets, H. De Meyer, H. Naessens: On Rational Cardinality-based Inclusion Measures, Fuzzy Sets and Systems 128 (2002) pp. 168-183

[2] B. De Baets, H. De Meyer, H. Naessens: A Class of Rational Cardinality-based Similarity Measures, Journal of Computational and Applied Mathematics 132 (2001) pp. 51-69

[3] Dimiter Driankov, H. Hellendron, M. Reinfrank: An Introduction to Fuzzy Control, Springer-Verlag Berlin-Heidelberg-NewYork, 1996, ISBN 3-540-56362-8

[4] J. Fodor, M. Rubens: Fuzzy Preference Modelling and Multicriteria Decision Support. Kluwer Academic Pub.,1994

[5] Fodor, J. C., De Baets, B., Calvo, T. (2003), Structure of uninorms with given continuous underlying t-norms and t-conorms, 24th Linz Seminar on Fuzzy Sets, 2003

[6] Rudas, Imre, Kaynak, O: New Types of Generalized Operations Computational Intelligence, Soft Computing and Fuzzy-Neuro Integration with Applications, Springer NATO ASI Series. Series F, Computer and Systems Sciences, Vol. 192, 1998, (O. Kaynak, L. A. Zadeh, B. Turksen, I. J. Rudas editors), pp. 128-156

[7] B. de Baets, J. Fodor Residual operators of uninorms, Soft Computing 3, 1999, 89-100, Springer-Verlag 1999

[8] Takacs, M., Approximate Reasoning in Fuzzy Systems Based on Pseudo-Analysis, Phd Thesis, Univ. of Novi Sad, 2004