Robust, Potential Limited Control for Systems of Unmodeled Internal Degrees of Freedom

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Abstract: In this paper robust Variable Structure / Sliding Mode control of a 2 Degrees Of Freedom (DOF) Classical Mechanical System, a ball-beam system is considered. The control task has the interesting feature that only one of the DOFs of the system, i.e. the position of the ball is controlled via controlling the other axis, the tilting angle of the beam. Since the acceleration of the ball rolling on the beam depends on the gravitation and the tilting angle of the beam, and due to the phenomenology of Classical Mechanical Systems the directly controllable physical quantity is the rotational acceleration of the beam, this system is a 4^{th} order one because it is the 4^{th} time-derivative of the ball's position that can directly be influenced by the control. Another interesting feature of this system is its "saturation" since the rotational angle of the beam must be limited within the interval (-90°, +90°) that also sets limits to the available acceleration of the ball. In the present approach a feedback control is applied in which the above limitation is achieved by the application of an angular potential and an angular velocity potential. The here applied robust control is based on the traditional concept of "error metrics".

Keywords: Robust Control; Variable Structure / Sliding Mode Control; Chattering reduction

1 Introduction

The accurate control of the "ball-beam system" in which a ball can roll on the surface of a beam the tilting angle of which is driven by some actuator is a physically interesting task. The motion of the ball essentially is determined by the tilting angle and the force of gravitation. This means that even if we are in the possession of a very strong actuator, the acceleration of the ball along the beam is limited by the above two factors. Since the directly controllable quantity is the torque determining the 2nd time-derivative of the angle tilting the beam, this system acts as a 4th order system in the sense that the 4th time-derivative of the ball's position along the beam is determined by the tilting torque. In more details it can be written that if the parameters of this system are as follows: the momentum of the beam Θ_{Beam}=2 kg×m^2, the mass of the ball m_{Ball}=2 kg, the radius of the ball r=0.05 *m,* and the gravitational acceleration is g=9.81 m/s^2. Via introducing the quantities $A=\Theta_{Beam}$, and $B=\Theta_{Ball}/r^2+m_{Ball}$, the following equations of motion are obtained:

$$\begin{aligned} A\ddot{\varphi}+m_{Ball}x\cos\varphi-m_{Ball}r\sin\varphi=0 \\ B\ddot{x}+m_{Ball}g\sin\varphi=0 \end{aligned} \quad . \tag{1}$$

in which variable φ describes the rotation of the beam counter-clockwisely with respect to the horizontal position in *rad* units, and x in m units denotes the distance of the ball from the center of the beam where it is supported. Variable Q [$N\times m$] describes the torque used for rotating the beam. From (1) d^2x/dt^2 can be expressed as a function of φ. Since this angle cannot be made abruptly vary, following two derivations by time d^4x/dt^4 can be expressed with $d^2\varphi/dt^2$ as follows:

$$x^{(4)}=\frac{m_{Ball}g}{B}\left(\sin\varphi\dot{\varphi}^2-\cos\varphi\ddot{\varphi}\right). \tag{2}$$

In the possession of the desired d^4x/dt^4 value, the dynamic model of the system, by the use of (2) and the 1st equation of the group (1) the necessary torque Q can be computed. If we have an exact system model, this computation corresponds to:

$$Q=\frac{-ABx^{(4)^{Des}}}{m_{Ball}g\cos\varphi}+A\tan\varphi\dot{\varphi}^2+xm_{Ball}g\cos\varphi-rm_{Ball}g\sin\varphi\ . \tag{3}$$

It is evident that if $\varphi\in(-\pi,\pi)$ then $\partial Q/\partial x^{(4)}<0$ that corresponds to a well defined control action according to which to increase $x^{(4)}$ Q has to be decreased, and vice versa. Normally it can be supposed that the parameters of the actual system are not precisely known. Instead of the actual parameters some *model values* are used as $\tilde{A}$, and $\tilde{B}$ constructed of the model values of the other parameters. On the basis of this *rough model* at first the desired rotational acceleration of the beam is estimated as

$$\ddot{\varphi}^{Des} = \frac{\widetilde{B}x^{(4)}}{\widetilde{m}_{Ball}\, g\cos\varphi} + \tan\varphi\dot{\varphi}^2 - \Gamma_{\varphi}\frac{\partial\cosh^{2n}\left(\frac{\beta\varphi}{1.5}\right)}{\partial\varphi} - \Gamma_{\dot{\varphi}}\frac{\partial\cosh^{2n}\left(\frac{\beta\dot{\varphi}}{1.5}\right)}{\partial\dot{\varphi}} \tag{4}$$

in which the last two terms two potentials are introduced for the *rotational angle* and for the *rotational velocity* of the beam. In the simulations β=5 [*dimless*], and n=1 [*dimless*] were used. The physical meanings of the potential terms are evident: the 1st term nonlinearly curbs the increase in $|\varphi|$ around 1.5 *rad*, the 2nd term curbs the angular velocity $|d\varphi/dt|$ around 3 *rad/s*. Nearby the small $|\varphi|$ angles and small $|d\varphi/dt|$ values the effects of these "flat" potentials are negligible. This "moderated" value for $|d^2\varphi/dt^{2Des}|$ is then substituted into the "model variant" of the 1st equation of the group (1) to calculate the necessary torque, Q. From practical point of view these potentials have the advantage that they are not singular but grow drastically nearby the appropriate limit values. By properly setting the values of the Γ parameters and β for the actual feedback policy these terms can guarantee that φ remains within certain limits. In the sequel the feedback policy applied in this control is discussed.

2 The Robust VS/SM Control

In the case of *robust Variable Structure / Sliding Mode Controllers* it is a popular choice to introduce the operator $(d/dt+\lambda)^{m-1}$ and apply it to the trajectory tracking error if the order of the set of differential equations determining the state propagation is m (in our case m=4) [1]:

$$S := \left(\frac{d}{dt} + \lambda\right)^{m-1}\left[x^{Nom} - x\right] \tag{5}$$

in which λ>0. If S=0 then $(d/dt+\lambda)^{m-2}\rightarrow 0$ exponentially. Roughly speaking it can be stated that during the time $\approx 2\lambda$ this quantity practically becomes 0. If this situation is achieved the term $(d/dt+\lambda)^{m-3}\rightarrow 0$ exponentially, etc. Via following this argumentation it can be expected that after finite time the tracking error $[x^{Nom}-x]$ starts to converge to zero exponentially. Since by calculating the time-derivative of S in (5) $x^{(m)Des}$ can be determined, in the typical case of *Robust Controllers* an *approximate system model* used to be satisfactory to drive S into the vicinity of 0 during finite time. For this purpose various strategies can be described. In this paper we try to prescribe the

$$\dot{S} = -K\mathrm{sign}(S) \tag{6}$$

strategy with positive constant K value. In the sequel this control is investigated via simulation. For the error metrics $\lambda=6/s$ was chosen in the forthcoming parts.

3 Simulation Results – Setting the Parameters

In the following simulations instead of the actual parameters the *model values* as $\tilde{A} = 2A$, $\tilde{m}_{Ball} = 0.4m_{Ball}$, $\tilde{\Theta}_{Ball} = 0.5\Theta_{Ball}$, $\tilde{B} = \tilde{\Theta}_{Ball} / r^2 + \tilde{m}_{Ball}$ were used. In Fig. 1 the formation of the phenomenon called "chattering" can be traced as the parameter K increases.

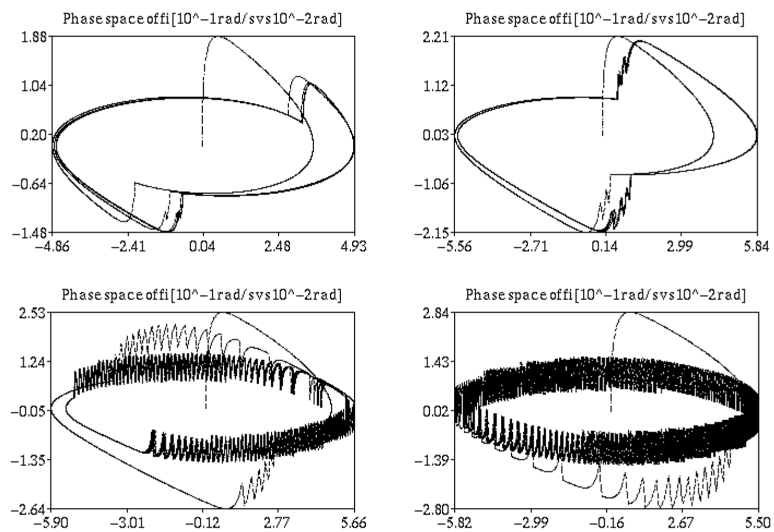

Figure 1

The formation of the phenomenon called chattering in the phase space of the tiliting angle of the beam ($d\varphi/dt$ [*rad/s*] vs. φ [*rad*] with increasing parameter K=10 (upper left), 20(upper right), 30 (lower left), and 40 (lower right) [m/s^4]

As Fig. 2 illustrates it the accuracy of the trajectory tracking significantly is improved with increasing K. This improvement can also be traced in the phase space of x, too. Since the fluctuations present in Fig. 1 appear in the 4th time-derivative of x, the phase-space of x remains smooth, these fluctuations are integarted out in it.

It also is an interesting task to check the modeling accuracy in the trajectory tracking of the robust controller. As it was mentioned (6) can be "exactly" realized only in the possession of the exact dynamic model of the system. From this point of view it can be expected that a considerable overestimation of the parameter A leads to more hectic motion due to overactuation effects. Leaving the other parameters of the control uncahnged, in Fig. 3 the $\tilde{A} = 4A$, $\tilde{A} = 2A$, $\tilde{A} = 0.5A$ estimated values were used for a slightly damped nominal trajectory. The decrease in the amplitude of chattering as well as the improvement of the tracking accuracy are evident on the basis of Fig. 3.

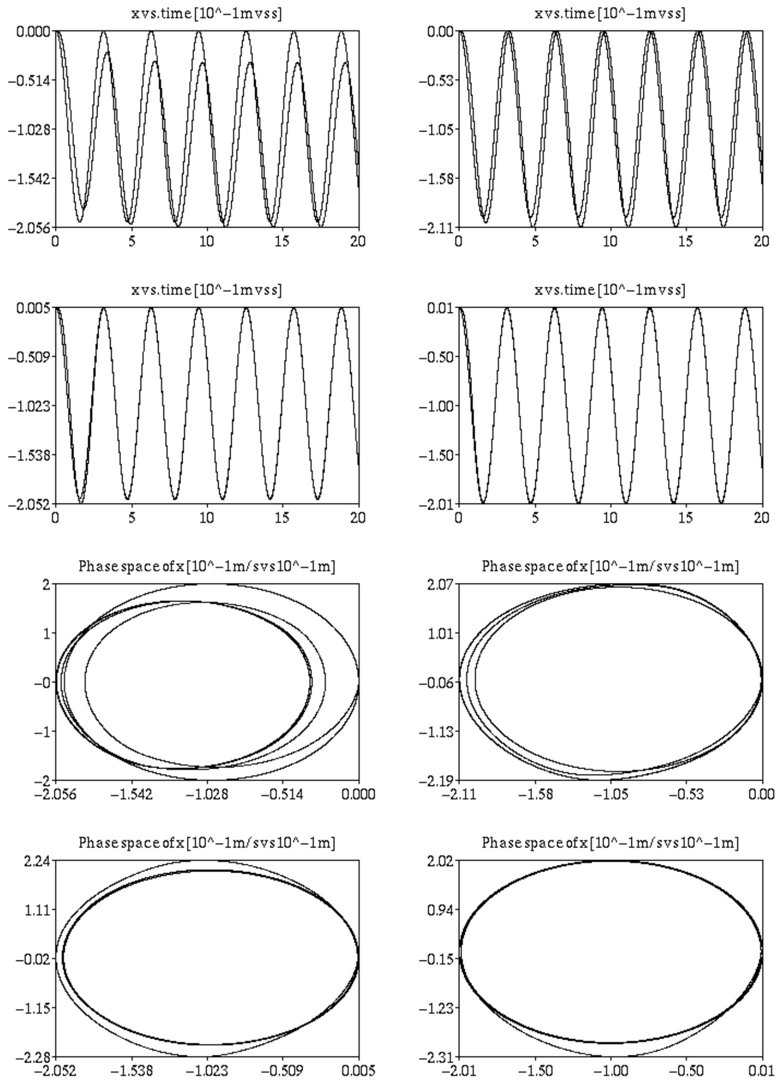

Figure 2

The improvement in trajectory tracking (*x* [*m*] vs.time [*s*]) with increasing parameter K=10 (1st raw left), 20 (1st raw right), 30 (2nd raw left), 40 (2nd raw right) [m/s^4], and the appropriate phase trajectories of the displacement of the ball (dx/dt [m/s] vs. x [m] in raws 3-4, respectively. In the last raw the great difference occurs at the initial error relaxation.

Of course besides manipulating the model parameters chattering can be reduced by simply smooting the switching rule in (6) by introducing a widht parameter *w* as e.g. in (7). Normally this smoothing reduces the tracking accuracy.

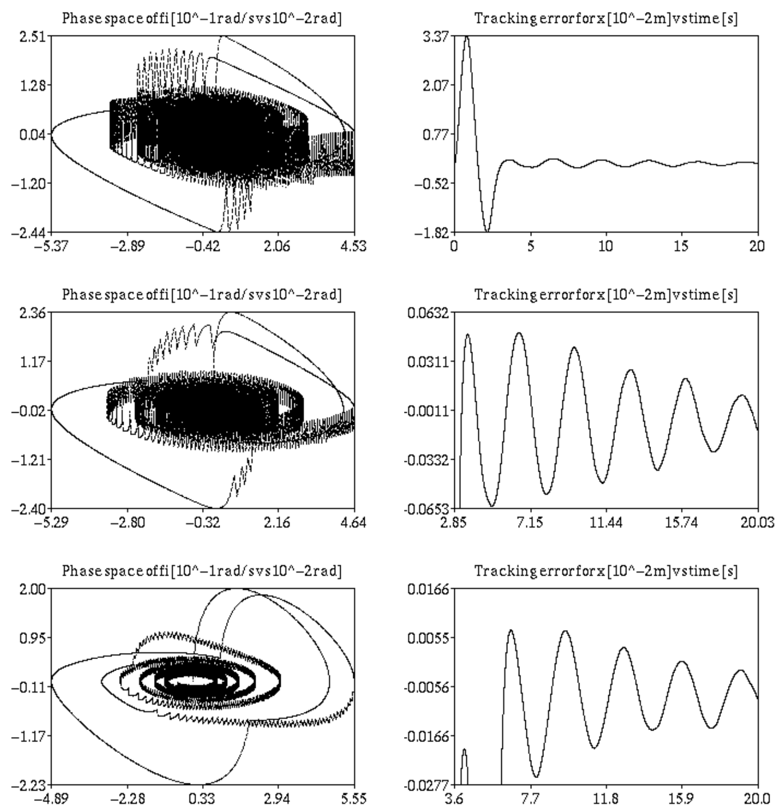

Figure 3

The reduction of chattering (the phase space of tilting angle $d\varphi/dt$ [*rad*] vs. φ [*rad*], 1st column) and trajectory tracking error (x^{Nom}-x) [*m*] vs.time [*s*], 2nd column) of the decrease in parameter $\widetilde{A} = 4A$ (1st raw), $\widetilde{A} = 2A$ (2nd raw), $\widetilde{A} = 0.5A$ (3rd raw). (In the last two rows of column 2 zoomed excerpts are displayed.)

$$\dot{S} = -K \tanh(2S / w) \tag{7}$$

In Fig. 4 the effect of the smoothing width *w* is described for two different values (w=6 and w=1) for the same system.

As it can well be seem increasing width *w* smooths the phase trajectory of the tilting angle φ, and, as a consequence, makes the relaxation of the error metrics slower. Its consequences also manifest themselves in the less accurate phase trajectory tracking, trajectory tracking for *x*, as well as allowing higher absolute values for the error metrics *S*.

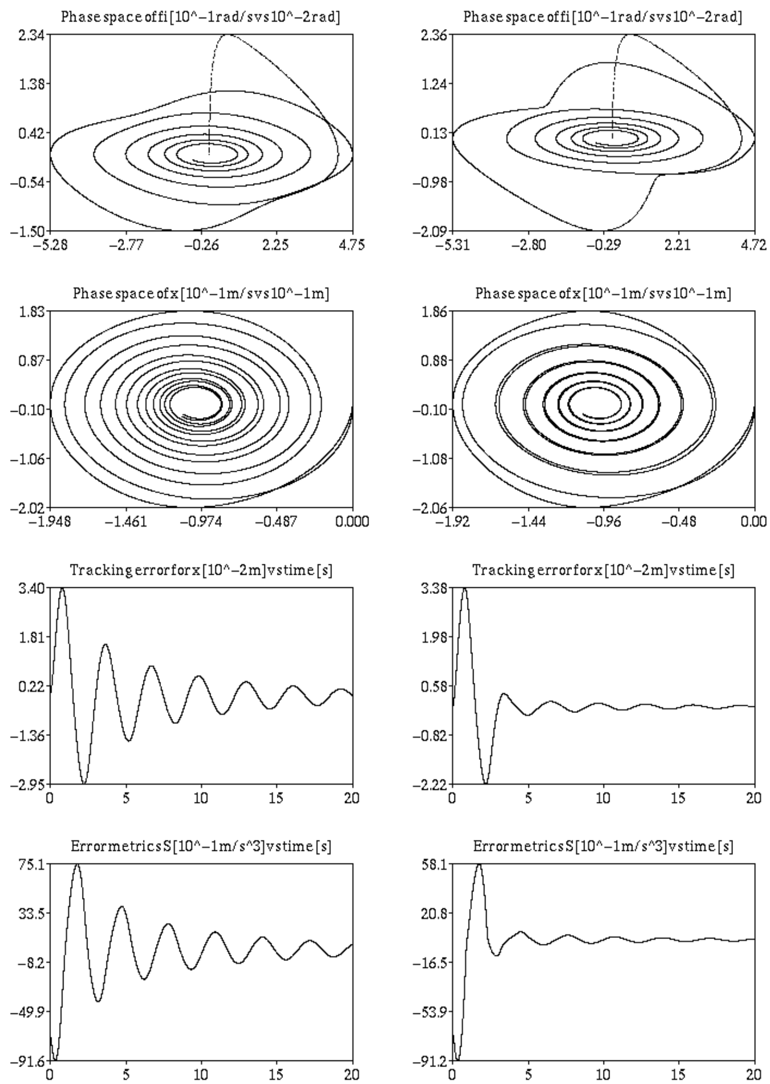

Figure 4

The effect of the smoothing width *w* on the control: *w*=6 (left column), *w*=1 (right column): the phase space of tilting angle $d\varphi/dt$ [*rad*] vs. φ [*rad*] (1st raw), the pahse space of the ball's displacement dx/dt [*m/s*] vs. *x* [*m*] (2nd raw), the trajectory tracking error (x^{Nom}-x) [*m*] vs.time [*s*] (3rd raw), and the error metrics *S* [m/s^3] vs. time [*s*] (4th raw)

Conclusions

In this paper a Variable Structure / Sliding Mode controller was developed or a 4^{th} orer physical system, the ball-beam system, in which the control task consists is positioning the ball by properly tilting the beam. While such control taks in general are considered on the basis of Barbalat's lemma and the Lyapunov function technique [1], [2], in our case simple, angular, and angular velocity potentials can limit the angular position an velocity of the tilted beam. These potentials behave in similar way as the functions of type κ, with the exception that their support is not bounded. The almost exponential increase of the here used function can compensate the effect of any linear feedback of fnite constant gains. The effects of the errors of the modeling parameters as well as compensation of chattering were successfully studied. The general expectation that at the costs of a little degradation of the tracking accuracy smooth control ca be devloped was confirmed by these investigations.

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