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Vague Environment-based Two-Step Fuzzy Rule Interpolation Method

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Abstract: The aim of this paper is to introduce a novel two-step Fuzzy Rule Interpolation Technique (FRIT) 'VEIN', based on the concept of Vague Environment. The strength of FRIT against classical fuzzy reasoning methods is the ability of gaining conclusion even in case where the knowledge is represented by sparse fuzzy rule bases. The FRIT 'VEIN' introduced in this paper is following the structure the Generalized Methodology of fuzzy rule interpolation [1], by adapting the concept of Vague Environment [4] for approximate description of fuzzy partitions [6].

Keywords: Vague Environment, Fuzzy Rule Interpolation, Fuzzy Set Interpolation, SingleŰ Rule Reasoning

1 Introduction

The conventional fuzzy rule based systems applying either the Zadeh-Mamdani-Larsen [14][9][8] concept or the Takagi-Sugeno [13] approach require the dense character of the rule base. Having a sparse rule base they cannot fire any of the rules for some observations, which do not overlap any of the rule antecedents at least partially. In such cases the fuzzy system based on them cannot produce an acceptable output. The problem can be solved by the application of Fuzzy Rule Interpolation Techniques (FRITs).

Since 1991 numerous FRITs have been proposed. They can be divided into two main groups. The first group produces the approximated conclusion from the observation directly; therefore its members are called one-step methods. The members of the second group reach the target in two steps. In the first step they interpolate a new rule whose antecedent part overlaps the observation at least partially. Then in the second reasoning step the estimated conclusion is determined by a Single Rule Reasoning (revision) method based on the similarity of the observation and the antecedent part of the newly interpolated rule. The main structure of the 'two-step' methods are summarised in the 'Generalized Methodology (GM) of fuzzy rule interpolation' introduced in [1] by Baranyi et al.

In this paper the authors propose a novel two-step FRIT called VEIN, based on the concept of Vague Environment. This concept, originally introduced by Klawonn in [4], is based on the similarity or indistinguishability of elements in a universe. Two values in a Vague Environment are ε-distinguishable if their distance is greater than ε, where the distances are weighted distances. The weighting factor or function is called *scaling function (factor)* [4]. Hence the Vague Environment can be characterized by its scaling function. There are ways for finding relation between the Vague Environment and fuzzy partitions see e.g. in [4], [6]. Therefore the concept of Vague Environment can be also applied for fuzzy reasoning, even for FRIT too. E.g. a one-step FRIT method (later named as 'FIVE') is introduced in [6].

2 Vague Environment-based Two-Step Fuzzy Rule Interpolation Method

The Vague Environment based two-step fuzzy rule INterpolation (VEIN) method essentially follows the concepts laid down by the Generalized Methodology (GM) of fuzzy rule interpolation [1]. It calculates the conclusion in two steps. It

1 interpolates a new rule in the position of the observation,

2 determines the conclusion by firing the new rule.

For the facilitation and simplification of the calculations all input and output partitions are normalized to the unit interval. The position of the linguistic terms is defined by its reference points. We use the centre of the core for this task. The distance between the sets is expressed by the mean of the Euclidean distance of their reference points.

In the first step the new rule is determined in three stages. These are the followings.

a) The shape of the antecedent sets is determined using a set interpolation method called VESI, which is based on the concept Vague Environment and presented more details in Section 3.

b) The position of the consequent sets is calculated by a crisp interpolation technique presented in Section 4.

c) The shape of the consequent sets is calculated using the same set interpolation technique as in stage a.

The final conclusion is determined in the second step of VEIN applying a special single rule reasoning method called REVE that is also based on the concept Vague Environment. Section 5 introduces REVE in details.

3 VESI

The method VESI (Vague Environment based Set Interpolation) aims the determination of a new linguistic term in a specific point, called interpolation point, of a fuzzy partition. It can be applied for the determination of the antecedent and consequent sets of the new rule in the first step of any FRI technique, which follows the concepts of the GM [1]. The method is the same regardless of it is used in case of a rule premise or a rule consequent. It can be applied in both the cases of sparse and dense partitions.

Similar to the other set or rule interpolation/extrapolation/approximation techniques VESI assumes regularity between the linguistic terms of a fuzzy partition. The first step of the technique is the generation of the VE of the partition, which has to be done only once, before starting the fuzzy system based on it. Throughout the course of the repetitive reasoning steps the original VE is used in the calculations.

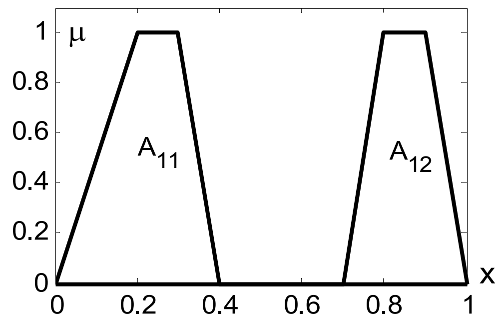

Figure 1
Sparse partition with two trapezoid shaped linguistic terms

The second step of VESI starts with a given interpolation point in the current dimension (partition), which is actually a reference point of a fuzzy set. The basic idea is that the new set conserves the properties of the VE. Let the prototypical point (where the membership value of the requested fuzzy set is equal to 1) be considered to be the interpolation point. Starting from it one can generate the shape of the new linguistic term easily from the scaling function.

Further on the algorithm is presented by a numerical example. For simplicity and lucidity the sample partition (Fig. 1) is sparse and it contains two trapezoid shaped CNF sets only. The sample linguistic terms are defined as follows

$$A_{11} = \{0.00/0, 0.20/1, 0.30/1, 0.40/0\}, \tag{1}$$

$$A_{12} = \{0.70/0, 0.80/1, 0.90/1, 1.00/0\}, \tag{2}$$

where the first subscript of the sets denotes the ordinal number of the input dimension and the second subscript indicates the ordinal number of the set in the current partition (dimension). The reference points of the sets are defined by the mid points of the cores ($RP(A_{11}) = 0.25$ and $RP(A_{12}) = 0.85$). In this example for the sake of simplicity the right flank of the set A_{11} and the left flank of the set A_{12} have the same slope in absolute value. The range of the partition is $R_{A_1} = [0, 1]$.

The scaling function builds up from the scaling factors (3)-(8) according to the membership shape of the element fuzzy sets

$$s_{A_{11}}^{L} = \left| \frac{d\mu_{A_{11}^{L}}}{dx} \right| = \frac{1}{0.20} = 5, \tag{3}$$

$$s_{A_{11}}^{C} = \left| \frac{d\mu_{A_{11}^{C}}}{dx} \right| = \frac{0}{0.10} = 0, \tag{4}$$

$$s_{A_{11}}^{R} = \left| \frac{d\mu_{A_{11}^{R}}}{dx} \right| = \frac{1}{0.10} = 10, \tag{5}$$

$$s_{A_{12}}^{L} = \left| \frac{d\mu_{A_{12}^{L}}}{dx} \right| = \frac{1}{0.10} = 10, \tag{6}$$

$$s_{A_{12}}^{C} = \left| \frac{d\mu_{A_{12}^{C}}}{dx} \right| = \frac{0}{0.10} = 0, \tag{7}$$

$$s_{A_{12}}^{R} = \left| \frac{d\mu_{A_{12}^{R}}}{dx} \right| = \frac{1}{0.10} = 10, \tag{8}$$

where s_X^Z is the scaling factor corresponding to the Z (left flank, right flank or core) part of the shape of the fuzzy set X (A_{11} or A_{12}).

To be conform to the extended concept of the VE [5][7] each interval of the scaling function delimited by prototypical points – including here also the sparse portions of the partition – is defined only by the neighboring flanks of the sets. Thus the interval [0.40, 0.70) is characterized by the same scaling factor $s=10$ as its surrounding intervals [0.30, 0.40) and [0.70, 0.80). In a similar way in case of the leading and trailing empty (sparse) portions of the partition the scaling factors of the closest set flanks are used for the generation of the VE. This feature ensures the capability of interpolation and extrapolation for the VE. Figure 2 presents the scaling function of the partition presented in Fig. 1.

Let we suppose that the interpolation point is given by the abscissa 0.50 and we intend to generate a normal ($height(A_1^i) = 1$) fuzzy set. The reference point of the new linguistic term will be identical with this point $RP(A_1^i)=0.50$. Thus the membership value of this point is $\mu_{A_1^i}(RP(A_1^i))=1$. The shape of the new set is calculated by integrating the scaling function. The left and right flanks are calculated separately. The left flank is determined point-wise starting from the reference point and proceeding towards *0* (the lower endpoint of the range of the linguistic variable) by applying the formula

$$\mu_{A_1^i}^{L}(x)=\max\left\{0,1-\int_{x}^{RP(A_1^i)}s(x)dx\right\}. \tag{9}$$

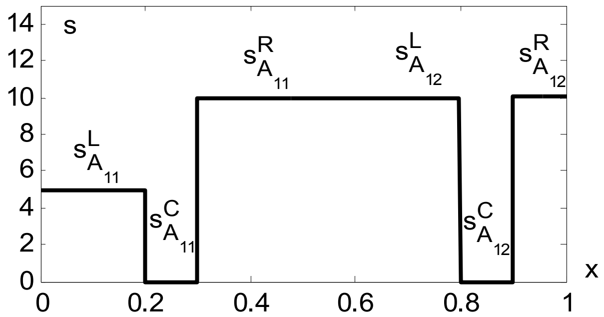

Figure 2
Vague Environment of the partition

The right flank is determined in a similar way only the endpoints of the interval and the direction are changed

$$\mu_{A_1^i}^{R}(x)=\max\left\{0,1-\int_{RP(A_1^i)}^{x}s(x)dx\right\}. \tag{10}$$

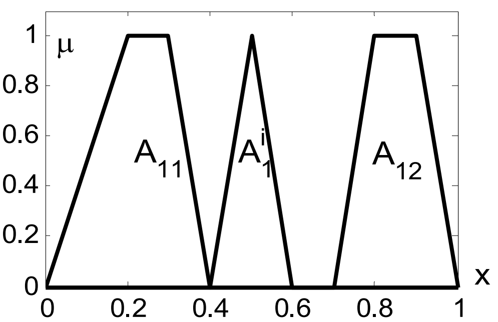

Figure 3
The resulting partition after the interpolation

In this case one is proceeding towards *1* (the upper endpoint of the range of the linguistic variable). By both sides the points are calculated using a loop construct. The termination condition is either the zero membership value of the current point or the reaching of an endpoint of the range of the linguistic variable.

In our example the resulting set is triangle shaped with the characteristic points situated at {0.40, 0.50, 0.60}. The partition containing the interpolated linguistic term is presented on Figure 3.

VESI fulfils most of the requirements defined in [3] as General conditions on FRI techniques. Some relevant features are listed below:

- The new set never can be an abnormal one due to the nature of the VE and to the algorithm that calculates the two flanks separately and joins them in the prototypical point.
- VESI preserves the ‘in between’ feature because the reference point of the new set is identical with the point of the interpolation.
- The condition ‘compatibility with the rule base’ is only fulfilled when the neighboring flanks have the same slope in absolute value.
- The fuzziness of the result depends on the shape of the neighboring flanks of the surrounding sets only in the simplest case (see Figs. 1 and 3). Having an interpolation point in the closest neighborhood of the reference point of a linguistic term the other flank of the sets also exercises influence on the calculations. Therefore the shape of the obtained set can contain some break-points.
- As a consequence of the above described feature VESI not always conserves the piece-wise linearity of the surrounding sets.
- FRI methods based on the proposed VESI can be easily extended to handle multidimensional antecedent universes as well.

4 Position of the Consequent Sets

The position of each consequent set of the new rule is identical with the reference point of the final conclusion in the respective dimension. They are calculated independently in the same manner each output dimension. Henceforth in the notations of the equations only the k^{th} dimension will be presented.

The basic assumption of the calculation is that each rule can be viewed as a point on a hyper-surface. The co-ordinates of this point are the reference points of the linguistic terms of the rule antecedent ($RP(A_{jm})$) and the reference point of the rule consequent ($RP(B_{km})$).

Thus the problem of finding the position of the consequent set of the interpolated rule can be reduced to a crisp n_a dimensional interpolation of irregularly spaced data, where n_a is the number of antecedent dimensions. Due to its easy applicability we use the extended version of the Shepard interpolation [11] for this task [2].

It calculates the demanded abscissa as a weighted average of the reference points of the consequent sets that belong to the known rules

$$RP\left(B_k^i\right) = \frac{\sum_{m=1}^{N} RP\left(B_{km}\right) \cdot w_m}{\sum_{m=1}^{N} w_m}, \tag{11}$$

where $RP\left(B_k^i\right)$ is the reference point of the consequent set of the interpolated rule in the k^{th} output dimension, N is the number of the rules, B_{km} is the consequent linguistic term of the m^{th} rule in the k^{th} dimension and w_m is the weight attached to the m^{th} rule.

The weighting is distance dependent. The position of each rule antecedent and also the position of the observation in the antecedent hyper-space can be characterized by points defined by the RPs of the linguistic terms used as co-ordinates. The applied weighting factor uses the Euclidean distances between these points

$$w_m = \frac{1}{\left(d\left(RA_m, A^*\right)\right)^2} = \frac{1}{\sum_{j=1}^{n_a} \left(RP\left(A_{jm}\right) - RP\left(A_j^*\right)\right)^2}, \tag{12}$$

where RA_m is the antecedent of the m^{th} rule, A^* is the observation, A_{jm} is the antecedent linguistic term of the m^{th} rule in the j^{th} input dimension, A_j^* is the observation in the j^{th} dimension.

5 REVE

The conclusion is calculated in the second step of the method by firing the interpolated rule. The antecedent part of the new rule does not fit always perfectly the observation. Therefore a revision based single rule reasoning method is applied, which calculates the conclusion by modifying the consequent sets of the new rule. This modification is related to the differences between the rule antecedent sets and the observation sets.

REVE (Revision mEthod based on the Vague Environment) is a novel single rule reasoning method and is also based on the concept Vague Environment. It is suggested as a complementary of the fuzzy set interpolation method VESI. However, it can be also applied as single rule reasoning technique in case of any two-step fuzzy rule interpolation method that follows the concepts of GM [1].

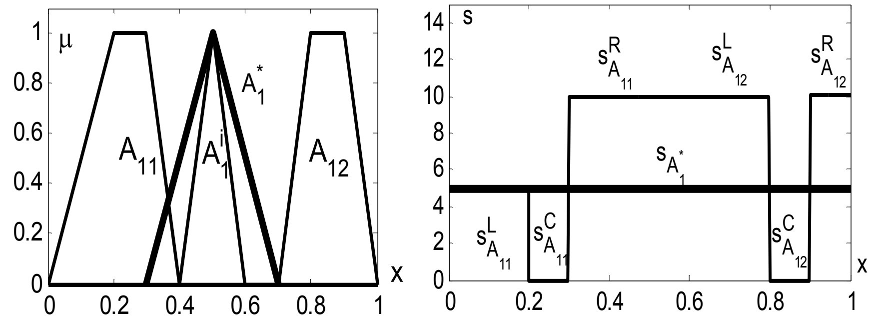

Figure 4
One dimensional antecedent universe of discourse, observation (A^* - bold line), interpolated antecedent set (A^i) and the corresponding scaling functions

The main idea of REVE is the conservation of the scaling function ratio in single rule reasoning. Having Vague Environments (and hence scaling functions) on both the rule antecedent and consequent sides, the scaling function ratio between the rule antecedent and the observation should be equal to the scaling function ratio between the rule consequent and the demanded conclusion. In other words, the similarity of fuzzy sets is expressed in the form of the similarities of the corresponding Vague Environments, in their scaling function ratio. For multidimensional antecedent universes, the basic ‘scaling function ratio’ idea could be simply extended to ‘mean scaling function ratio’ as well.

REVE first calculates for each input dimension the ratio of the scaling function describing the Vague Environment of the observation ($s_{A_j^*}(x)$, see Fig. 4) and the scaling function of the antecedent partition ($s_{A_j}(x)$)

$$r_{A_j}(x) = \frac{s_{A_j^*}(x)}{s_{A_j}(x)}, \tag{13}$$

where *j* is the number of the current antecedent dimension. Further on for a simple demonstration of the calculations we consider a SISO (Single Input Single Output) fuzzy system, which is defined as follows. Its antecedent universe is described by (1) and (2) (see Fig. 1). The consequent universe is given by (14) and (15) (see Fig. 5). The rule base contains two rules: $A_{11} \rightarrow B_{11}$ and $A_{12} \rightarrow B_{12}$. The observation is triangle shaped and is defined by (16) (see Fig. 4). The reference point of the conclusion ($RP(B_1^*) = 0.4027$) was calculated by the extended version of the Shepard interpolation.

$$B_{11} = \{0.15/0, 0.20/1, 0.25/0\} \tag{14}$$

$$B_{12} = \{0.75/0, 0.80/1, 0.85/0\} \tag{15}$$

$$A_1^* = \{0.30/0, 0.50/0, 0.70/0\} \tag{16}$$

Thus the ratio of the scaling functions is

$$r_{A_1}(x) = \begin{cases} 5/5 = 1, & x \in [0, 0.2) \\ 5/0 = \infty, & x \in [0.2, 0.3) \vee x \in [0.8, 0.9) \\ 5/10 = 0.5, & x \in [0.3, 0.8) \vee x \in [0.9, 1] \end{cases} \tag{17}$$

Next the harmonic mean of the antecedent ratios is calculated

$$mr_A(x) = \frac{n_a}{\sum_{j=1}^{n_a} \frac{1}{r_{A_j}(x)}}, \tag{18}$$

where n_a is the number of antecedent dimensions. In our example $mr_A(x) = r_{A_1}(x)$. The rest of the calculations are done separately for each consequent dimension. Further on the case of the k^{th} output dimension is considered. In the example $k=1$.

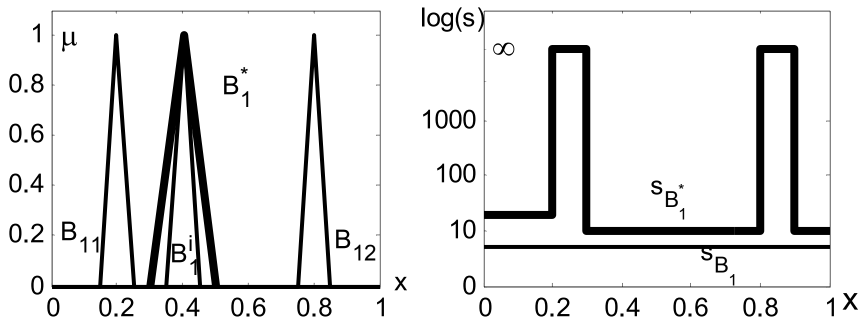

Figure 5

One dimensional consequent universe, interpolated consequent set (B_1^i), conclusion (B_1^* -bold line), and the corresponding scaling functions

The basic idea of REVE is the principle of the conservation of the mean scaling function ratio. Thus the scaling function inducing the Vague Environment of the conclusion is determined by considering the same ratio between the scaling functions of the conclusion and the consequent partition as the mean scaling function ratio calculated on the antecedent side

$$r_{B_k}(x) = mr_A(x) = \frac{s_{B_k^*}(x)}{s_{B_k}(x)}, \tag{19}$$

where $s_{B_k}(x)$ is the scaling function of the k^{th} consequent dimension, and $s_{B_k^*}(x)$ is the scaling function describing the conclusion in the k^{th} dimension (see Fig. 5). The scaling function of the conclusion results from the formula (19) as follows

$$s_{B_k^*}(x) = s_{B_k}(x) \cdot mr_A(x) \cdot \tag{20}$$

Thus the example application leads to the function (21), as it is demonstrated in Figure 5.

$$s_{B_1^*}(x) = \begin{cases} 20, & x \in [0, 0.2) \\ \infty, & x \in [0.2, 0.3) \vee x \in [0.8, 0.9) \\ 10, & x \in [0.3, 0.8) \vee x \in [0.9, 1] \end{cases} \tag{21}$$

The right side of Fig. 5 presents the scaling function of the conclusion calculated for the whole range [0, 1]. This is done for illustration purposes, in practical applications it has to be calculated only for that portion of the universe of discourse, which is necessary for the determination of the shape. The membership function is calculated similar to the example presented in the section describing VESI (see (9) and (10)). It is triangle shaped with characteristic points at {0.3027, 0.4027, 0.5027}.

The method REVE has low computational complexity and its effectiveness increases with the number of consequent dimensions. It is due to the fact that the calculations on the antecedent side have to be done only once and the conclusion sets are determined separately in each consequent dimension. Thus several steps can made parallel if the computing environment it enables. The computational needs can be reduced further when the method is applied together with VESI by using the same pregenerated antecedent and consequent Vague Environments.

The adaptation of the harmonic mean ensures that the conclusion will be crisp ($s_{B_k^*}(x) = \infty$) if and only if all observation sets are crisp ($s_{A_j^*}(x) = \infty \ \forall j$). The condition on compatibility with the rule base introduced in [3] is also fulfilled (if the scaling functions of the observation and the rule antecedent are the same in all dimensions the conclusion will be equal to the rule consequent). Due to the ratio based conservation principle, the direction of the fuzziness changing of the conclusion follows the direction of the fuzziness changing of the observation. Owing to these two features the less uncertain the observation is the less fuzziness will have the approximated conclusion.

Another advantage of the proposed REVE method is the lack of verification and correction steps required in many other methods for gaining valid, convex and normal fuzzy conclusion, hence from the method it is straightforward, that the conclusion of REVE is always a valid convex and normal fuzzy set.

The main disadvantage of REVE is the lack of compatibility with the rule base in case, when only an approximate scaling function can be determined in any of the antecedent or consequent partitions. In addition the piece-wise linearity is also not always conserved.

Conclusions

FRITs offer a suitable solution for handling fuzzy systems where the knowledge is represented by a sparse fuzzy rule base. This paper introduced a novel two-step FRIT called VEIN, based on the concept of Vague Environment. As an additional benefit of the proposed method, VEIN fulfils most of the FRIT conditions suggested in [3].

The main advantage of the proposed method is its quick inference process, which ensures the real-time applicability as well. As a main drawback, the lack of compatibility with the rule base in case of some membership function types when approximate scaling functions are required should be also mentioned. This topic is subject of further research work.

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