

## **Adaptive Control of a Differential Hydraulic Cylinder with Dynamic Friction Model**

**József K. Tar, Imre J. Rudas, János F. Bitó**

Institute of Intelligent Engineering Systems  
John von Neumann Faculty of Informatics  
Budapest Tech  
Bécsi út 96/B, H-1034 Budapest, Hungary  
tar.jozsef@nik.bmf.hu, rudas@bmf.hu

**Kazuhiro Kosuge**

Department of Machine Intelligence and Systems Engineering  
Graduate School of Engineering  
Tohoku University  
Aoba-yama 01, Sendai 980-8579, Japan  
kosuge@irs.mech.tohoku.ac.jp

*Abstract: Servo valve controlled hydraulic differential cylinders are non-linear, strongly coupled multivariable electromechanical systems. When the piston's velocity with respect to the cylinder is in the vicinity of zero the effect of adhesion is a significant effect due to which decreasing absolute value of velocity results in increasing stiction force. Furthermore, when the motion is initiated from equilibrium state of zero initial velocity adhesion can compensate arbitrary forces within certain limits keeping the piston almost fixed. This regime of friction can be modeled by some elastic deformation the deformable elements of which become disconnected over certain force limits. In the paper a concise application of the dynamic LuGre Model of friction is reported in which the effects of the elastic deformation, adhesion, and viscosity are combined with each other. To compensate the effect of the imprecisely known system parameters and unknown external forces an adaptive control is developed in which varying fractional order derivatives are used to reduce the hectic behavior of friction in the case of 'critical' trajectories that asymptotically converge to a fixed position and zero velocity. Simulation results made by INRIA's Scilab are presented. It is concluded that the combined application of the two adaptive techniques can result in accurate control if the LuGre model satisfactorily describes the reality.*

*Keywords: Adaptive Control; Nonlinear Control; Dynamic Friction; LuGre Friction Model*

## 1 Introduction

Hydraulic servo valve controlled differential cylinders are strongly coupled non-linear electromechanical devices of multiple parameters which are difficult to be kept under perfect control. The viscosity of the oil in the pipe system is very sensitive to the temperature that normally increases due to the friction in the circulation. Oil compressibility depends on the amount of air or other gases solved in it. Adhesion of the piston at the cylinder introduces rough nonlinearity into the behavior of the system. The combination of these effects with the not always measurable external disturbance forces can make a quite complex control task arise. Hydraulic cylinders also have a very important property: due to the compressibility of the working fluid and elastic deformation of the pipe system the pressures in its chambers cannot abruptly be changed. It is the time-derivative of the oil pressure related to the 3<sup>rd</sup> time-derivative of the piston's displacement can abruptly be prescribed. The hectic behavior of friction forces also are related to this 3<sup>rd</sup> derivative, that is the control has to be developed for ab ovo noisy signals.

The problem of driving a flexible robot arm under external disturbances by a hydraulic servo valve controlled differential cylinder was studied and solved in two alternative manners by Bröcker and Lemmen in [1]. Their first approach was based on the 'disturbance rejection principle', the other one on the 'partial flatness principle', respectively. In each case it was necessary to measure the disturbance force and its time-derivative as well as to know the exact model of the hydraulic cylinder they developed in details and identified for a particular robot arm-drive system. For describing friction they used the static Striebeck model. However, the identification of such a system needs a lot of laboratory work the result of which may also be temporal. A serious problem is the need for measuring the external disturbance forces, too. In general it seems to be expedient to apply adaptive control instead of trying to measure the ample set of unknown and time-varying parameters. This adaptive control need not to be too intricate, actually should not be much more complicated than an industrial PID controller. For this purpose Soft Computing (SC) based approaches would be more attracting than detailed analytical modeling.

Unfortunately traditional SC (both fuzzy systems, and neural networks) suffer from bad scalability properties: the number of either the network nodes or the fuzzy rules is drastically increasing function of the degree of freedom of the system to be controlled. In order to get rid of the scalability problems of the classical Soft Computing a novel approach was initiated that is based on a compromise between the need of generality and scalability in [2]. It was shown by the use of perturbation calculus that this method can be applied for a quite wide class of physical systems, e.g. in the case of Classical Mechanical Systems, too [3]. This approach uses far simpler and far more lucid uniform structures and procedures than the classical ones: various algebraic blocks originating from different Lie groups can be incorporated into the 'model'. In order to reduce noise-

sensitivity the approach described in this paper allows a  $PID^{var}$  control for the piston's desired trajectory in which the order of derivation depends on the past fluctuation of the piston's velocity that generates harsh modification in the friction forces. In the sequel the main point of the scalable soft computing is very briefly outlined. Following that the analytical model of the differential hydraulic servo cylinder based on the dynamic LuGre friction model is presented together with the new control approach applied. The paper is closed by the simulation results and the conclusions.

## 2 The Adaptive Control

The adaptive control used in this case is based on the concept of Complete Stability that often is used as a practical criterion for the controlled system. It means that for a constant input excitation the system's output asymptotically converges to a constant response. If the variation of the input is far slower than the system's dynamics, with a good accuracy, it provides us with a continuous response corresponding to some mapping of the time-varying input [4]. Therefore, let us suppose that there is given some imperfect model of the system on the basis of which some excitation  $\mathbf{e}$  is calculated to obtain a desired system response  $\mathbf{i}^d$  as  $\mathbf{e} = \boldsymbol{\varphi}(\mathbf{i}^d)$ . The system has its inverse dynamics described by the unknown function  $f()$  as  $\mathbf{i}^r = \boldsymbol{\psi}(\boldsymbol{\varphi}(\mathbf{i}^d)) = f(\mathbf{i}^d)$  resulting in a realized response  $\mathbf{i}^r$  instead of the desired one. Normally one can obtain information on  $f()$  only by observing its appropriate *input* and *output* values. In general this function can considerably vary in time. The control deforms the input of  $f()$  [ $\mathbf{i}^{d*}$ ] to achieve and maintain the  $\mathbf{i}^d = f(\mathbf{i}^{d*})$  state by applying a series of linear transformations defined as

$$\mathbf{i}_0; \mathbf{S}_1 f(\mathbf{i}_0) = \mathbf{i}_0; \mathbf{i}_1 = \mathbf{S}_1 \mathbf{i}_0; \dots; \mathbf{S}_n f(\mathbf{i}_{n-1}) = \mathbf{i}_0; \mathbf{i}_{n+1} = \mathbf{S}_{n+1} \mathbf{i}_n. \quad (1)$$

If this series converges to the identity operator ( $\mathbf{S}_n \rightarrow \mathbf{I}$ ) just the proper deformation is approached, therefore the controller 'learns' the behavior of the observed system via step-by-step amendments of the initial model. For making the problem mathematically unambiguous the ambiguous conditions in (1) can be transformed into matrix equations by putting the values of  $f$  and  $\mathbf{i}$  into well-defined blocks of bigger matrices as e.g.

$$\mathbf{S}_n \begin{bmatrix} \mathbf{f}_{n-1} & \dots \\ d & \dots \\ \dots & \dots \end{bmatrix} = \begin{bmatrix} \mathbf{i}^d & \dots \\ d & \dots \\ \dots & \dots \end{bmatrix} \Rightarrow \mathbf{S}_n = \begin{bmatrix} \mathbf{i}^d & \dots \\ d & \dots \\ \dots & \dots \end{bmatrix} \times \begin{bmatrix} \mathbf{f}_{n-1} & \dots \\ d & \dots \\ \dots & \dots \end{bmatrix}^{-1} \quad (2)$$

in which the dots (...) denote the other columns of the matrices that contain the arbitrary parameters of this ambiguous task. Parameter  $d$  is a 'dummy', that is physically not interpreted constant value having the role of evading the occurrence

of the mathematically dubious  $0 \rightarrow 0$ ,  $0 \rightarrow \text{finite}$ ,  $\text{finite} \rightarrow 0$  transformations. There is a lot of possibilities to construct easily invertible matrices in (2). For this purpose it is expedient to use elements of certain Lie groups that guarantees to find element in arbitrary vicinity of the unit matrix that is needed for the  $\mathbf{S}_n \rightarrow \mathbf{I}$  convergence. In [5] various algebraic methods were mentioned in connection with the adaptive control of hydraulic differential servo cylinders. In the present paper special symplectic matrices are used as

$$\mathbf{S} = \left[ \begin{array}{c|ccc} \mathbf{0} & -\frac{1}{s}\mathbf{m}^{(1)} & -\frac{1}{s}\mathbf{m}^{(2)} & -\mathbf{e}^{(3)} \dots -\mathbf{e}^{(n+2)} \\ \hline \mathbf{m}^{(1)} & \mathbf{m}^{(2)} & \mathbf{e}^{(3)} \dots \mathbf{e}^{(n+2)} & \mathbf{0} \end{array} \right] \quad (3)$$

$$\mathbf{M} = [\mathbf{m}^{(1)}, \mathbf{m}^{(2)}, \mathbf{m}^{(3)}, \dots, \mathbf{m}^{(n+2)}] = \begin{bmatrix} f_1 & -f_1 & e_1^{(3)} & \dots & e_1^{(n+2)} \\ \dots & \dots & \dots & \dots & \dots \\ f_n & -f_n & e_n^{(3)} & \dots & e_n^{(n+2)} \\ d & -d & e_{n+1}^{(3)} & \dots & e_{n+1}^{(n+2)} \\ D & \frac{\mathbf{f}^2 + d^2}{D} & e_{n+2}^{(3)} & \dots & e_{n+2}^{(n+2)} \end{bmatrix} \quad (4)$$

$$D^2 \equiv \mathbf{f}^T \mathbf{f} + d^2, s = 2D^2 \quad (5)$$

in which  $n$  denotes the degree of freedom of the system to be controlled. The first two columns of  $\mathbf{M}$  are trivially orthogonal to each other. Parameter  $d$  is the ‘dummy’ component used to evade singular transformations, and the symbols  $\mathbf{e}^{(3)}, \dots, \mathbf{e}^{(n+2)}$  denote orthogonal unit vectors that lie in the orthogonal sub-space of the first two columns of  $\mathbf{M}$ . They can be created e.g. by rotating a given vector  $\mathbf{b}$  to into the direction of another given vector  $\mathbf{a}$  while leaving the orthogonal sub-space of these vectors invariant. So if the operation starts with an orthonormal set  $\{\mathbf{e}^{(1)}, \dots, \mathbf{e}^{(n+2)}\}$  and at first it is rigidly rotated until  $\mathbf{e}^{(1)}$  becomes parallel with the 1<sup>st</sup> column of  $\mathbf{M}$ , its 2<sup>nd</sup> column will lie in the orthogonal subspace of the 1<sup>st</sup> one spanned by the transformed unit vectors  $\{\mathbf{e}^{*(2)}, \dots, \mathbf{e}^{*(n+2)}\}$ . In the next step this whole set can rigidly be rotated until the new  $\mathbf{e}^{** (2)}$  vector becomes parallel with the 2<sup>nd</sup> column of  $\mathbf{M}$ . This operation can be so constructed that it leaves the orthogonal subspace of these two vectors invariant. Since the previously determined  $\mathbf{e}^{*(1)}$  lies within this subspace, it remains invariant during this rotation. It is very easy to construct these rotations in closed analytical form requiring not too much computations.

In [5] the friction was modeled by the static Striebeck model. In the present approach it is replaced by the physically more complete and concise dynamic LuGre model that is discussed in details in the sequel.

### 3 Model and Control of the Hydraulic Differential Cylinder

The operation of the differential hydraulic cylinder was described in details e.g. in [1]. Let  $y$  denote the linear position of the piston in  $m$  units. The acceleration of the piston with respect to an inertial system of reference is given as

$$\ddot{y} = \frac{1}{m} \left[ \left( p_A - \frac{1}{\varphi} p_B \right) A_A - F_f(\dot{y}) - F_d \right] \quad (6)$$

in which  $p_A$  and  $p_B$  denote the pressures in chamber A and B of the piston in *bar*,  $\varphi = A_A/A_B$ , that is the ratio of the ‘active’ surfaces of the appropriate sides,  $m$  is the mass of the piston in *kg*,  $F_f$  denotes the internal friction acting between the piston and the cylinder,  $F_d$  denotes the external disturbance force. The pressure of the oil in the chambers also depends on the piston’s position and velocity as

$$\dot{p}_A = \frac{E_{oil}}{V_A(y)} (-A_A \dot{y} + B_v K_v a_1(p_A, \text{sign}(U)) U) \quad (7)$$

$$\dot{p}_B = \frac{E_{oil}}{V_B(y)} \left( \frac{A_A}{\varphi} \dot{y} - B_v K_v a_2(p_B, \text{sign}(U)) U \right) \quad (8)$$

where  $B_v$  denotes the flow resistance,  $K_v$  is the valve amplification,  $U$  is the *valve voltage*. (In [1] this quantity was considered as a ‘normalized’ quantity. In the present simulations it is supposed to be only bounded that makes it possible normalization in the case of any actual implementation.) The oil volume in the pipes and the chambers also depend on  $y$  and the cylinder stroke  $H$  [*m*] as

$$V_A(y) = V_{pipeA} + A_A y, \quad V_B(y) = V_{pipeB} + A_B (H - y) \quad (9)$$

The hydraulic drive has two stabilized pressure values, the *pump pressure*  $p_o$  and the *tank pressure*  $p_r$ . Under normal operating conditions (that is when no shock waves travel in the pipeline) these pressures set the upper and the lower bound to  $p_A$  and  $p_B$ . The functions  $a_1$  and  $a_2$  in (7) and (8) are strongly non-linear functions representing the general feature of strongly turbulent flow through pipes or holes that the necessary pressure difference between the input and the output sides is proportional to the square of the flow of velocity. This a kind of more or less ‘cubistic’ or rough approximation of the reality that has a more detailed description by the Moody Diagram [6] that also has a narrow band for the laminar regime and a stochastic one for the transition between the laminar and the turbulent regimes.

Furthermore, the system’s resistance slightly depends on the flow velocity even in the turbulent regime, too.

$$\begin{aligned}
 a_1(p_A, \text{sign}(U)) &= \begin{cases} \text{sign}(p_0 - p_A) \sqrt{|p_0 - p_A|} \\ \text{if } U \geq 0, \\ \text{sign}(p_A - p_i) \sqrt{|p_A - p_i|} \\ \text{if } U < 0 \end{cases} \\
 a_2(p_B, \text{sign}(U)) &= \begin{cases} \text{sign}(p_B - p_i) \sqrt{|p_B - p_i|} \\ \text{if } U \geq 0, \\ \text{sign}(p_0 - p_B) \sqrt{|p_0 - p_B|} \\ \text{if } U < 0 \end{cases}
 \end{aligned} \tag{10}$$

However, it is reasonable to suppose that in the case of the normally used hydraulic machines practically only the turbulent cases occur, and that the singularity of the square root function in zero well represents the drastic singularity of the reality whenever fast change in the flow direction of turbulent flows happens. Furthermore, under ‘normal conditions’  $\text{sign}(a_1) \geq 0$ , and  $\text{sign}(a_2) \geq 0$ , are realistic suppositions according to the limiting role of the pump and tank pressures for  $p_A$  and  $p_B$ .

For practically acceptable modeling of friction in (6) in the present paper the dynamic *LuGre Model* is used. Modeling and compensation of friction obtains attention even in the most recent works, too. In the special case of single variable systems efficient identification techniques were recently elaborated by Seung-Jean Kim *et al* [7], and L. Márton [8]. These techniques use certain friction models previously introduced by other researchers. The so called static models as the *Striebeck Model* approximating the stick-slip phenomena play important role in the low speed control regime as e.g. in [9]. This model establishes a simple approximate functional relationship between the relative velocity of the surfaces sliding on each other and the friction forces whenever this relative velocity  $v$  differs from zero as

$$F = -\text{sign}(v) \left[ F_c + F_s \exp\left(-\frac{|v|}{v_s}\right) \right], v \neq 0. \tag{11}$$

This model does not give satisfactory description of the ‘sticking’ phenomenon i.e. the observation that  $v$  stagnates at zero until the external force achieves an  $F_c + F_s (> F_c)$  limit in its absolute value. It describes only the experienced behavior that the friction force decreases with increasing absolute value of the velocity. For numerical simulation of stiction this model has to be ‘completed’ by the introduction of a fictitious small velocity region centered near zero. If this region is achieved the velocity is kept at exactly zero until the external force exceeds the limit  $F_c + F_s$ . This way of modeling is not very well substantiated on physical basis and the actual value of this velocity limit remains dubious. To provide some physical picture the so called *Tustin Model* can be introduced in which some

variable ‘ $z$ ’ describes a kind of internal deformation of the connected surfaces on which the friction force depends. It is a hidden internal degree of freedom that has its own equation of state propagation as

$$\frac{dz}{dt} = v - \frac{\sigma_0 |v|}{F_c + F_s \exp(-|v|/v_s)} z, \quad F = \sigma_0 z. \quad (12)$$

The simple picture behind this model is the supposition that some elastic deformation happens via small springs that partly are destroyed (disconnected) with higher displacements. Consequently  $z$  can be increased in its absolute value only to a velocity-dependent limit, and it stagnates at this value until the velocity changes its sign. This changing sign causes abrupt, discontinuous variation in  $dz/dt$ , and a fast variation in  $z$ . In the dynamic *LuGre Model* the above contribution is completed by a pure viscous term, and an additional one behind which the deformation of the bristles of some ‘brush’ are hidden as physical models:

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + F_v v \quad (13)$$

while (12) remains valid, too. In (13) the proper orientation has to be taken regarding the surfaces in contact.  $F_v$  describes the viscous friction coefficient, and  $\sigma_1$  is a new parameter pertaining to the effect of the bending bristles. This model is physically complete in the sense that no any ‘*velocity limit*’ of dubious interpretation must be introduced for its use. The behavior of the whole system is described by the dynamic coupling between the hidden internal and the observed degrees of freedom. On this reason in the present paper we used (13) in the simulations for estimating the effect of friction. In the rough model on the basis of which the control was developed this friction model was not taken into account at all.

Returning to the question of the rough model based control, the following considerations were done. For the tracking error  $e := (y^R - y^{Nom})$  a simple PID controller was constructed in the following manner:

$$\ddot{e} = -Pe - D\dot{e} - I \int_0^t e dt \quad (14)$$

The appropriate  $P$ ,  $D$ , and  $I$  coefficients were determined simply by substituting an expected  $e = \exp(\alpha t)$  type relaxation into the time-derivative of (14) that results in a third order polynomial for  $\alpha$ . For this polynomial three, slightly different negative real roots were prescribed in the form of  $(\alpha - \alpha_1)(\alpha - \alpha_2)(\alpha - \alpha_3)$ . Substituting this into (14)  $P$ ,  $D$ , and  $I$  can conveniently be determined. The time-derivative of (14) therefore leads to the *desired third time-derivative of the piston's trajectory* as

$$\ddot{y}^d = \ddot{y}^{Nom} - P\dot{e} - D\ddot{e} - Ie \quad (15)$$

The very rough approximate model of the cylinder was obtained by omitting the friction forces and the external disturbance forces in (6) as

$$\ddot{y}^d = \frac{A_A}{m} \left( \dot{p}_A - \frac{1}{\varphi} \dot{p}_B \right) \quad (16)$$

into which the desired time-derivative of the piston's acceleration was substituted and the desired value for  $dp_A/dt$  was set to 0. Eq. (16) thus immediately yields an 'expected' value for  $d(p_A-p_B)/\varphi/dt$ . Via computing  $[(7)-(8)/\varphi]$  this determines the proposed control signal  $U$ , and from the known current state of the system and (7) and (8) the actually obtained  $dp_A/dt$ , and  $dp_B/dt$  values can be computed. This can be substituted into the time-derivative of (6) yielding the 'actual' third time-derivative of the piston's displacement. Here special attention has to be paid to the problem of observing  $d^3y/dt^3$ , which, in the case of the presence of friction forces, may be critical. For filtering out the noisy part of this signal Caputo's definition of the fractional order derivatives can be applied. It *re-integrates* the integer order derivative with a kernel function of long tail acting as a frequency filter. According to that (15) can be modified as

$$y^{(2+\beta)^d} = \int_0^t d\tau \left[ \ddot{y}^{Nom}(\tau) - P\dot{e}(\tau) - D\ddot{e}(\tau) - Ie(\tau) \right] \frac{(t-\tau)^{-\beta}}{\Gamma(1-\beta)}, \quad \beta \in (0,1). \quad (17)$$

In the practical realization of that the lower limit of the integration is replaced by a finite memory  $t-T$ . In the numerical approximation of the integral with singular integrand the full interval of the integration of length  $T$  is divided into small ones of length  $\delta$  during which the reintegrated derivative is supposed to be approximately constant (details are given in e.g. in [12]). The next essential point is setting the order of derivation. Since according to (13) changing sign of the velocity generates drastic changes in the friction forces, due to the controller's feedback this force can oscillate whenever zero-transmission happens in the velocity. That is,  $\beta \approx 1$  is needed for non-zero velocities, and  $\beta < 1$  whenever the velocity is in the vicinity of zero. In the present paper the adaptive formula (18) was applied, in which instead of the velocity, the observed 3<sup>rd</sup> time-derivatives are used, because this signal is directly related to the controller's feedback.

$$0 < \beta = \left( A + \left| \sum_{s=1}^{T/\delta} \text{sign}(\ddot{y}(t-s\delta)) \right|^\gamma \right) / \left( A + (T/\delta)^\gamma \right) \leq 1. \quad (18)$$

In (18) there are various parameters as  $A$ ,  $T$ ,  $\delta$  and  $\gamma$  the actual values of which numerically concern the quality of control. In the simulations  $\delta=1$  ms was chosen as a fixed value. The other values were chosen as follows:  $A=1$ ,  $T=20$  ms, and  $\gamma=2.5 \times 10^{-4}$ . It was found that it is expedient to choose very sharp reduction of the order of derivation, so the above  $\gamma$  was found to be almost 'optimal'. (The actual value of  $A$  was not very important.) The role of the adaptive control was to correct the errors of this rough model and the consequences of the unmeasured external disturbances.

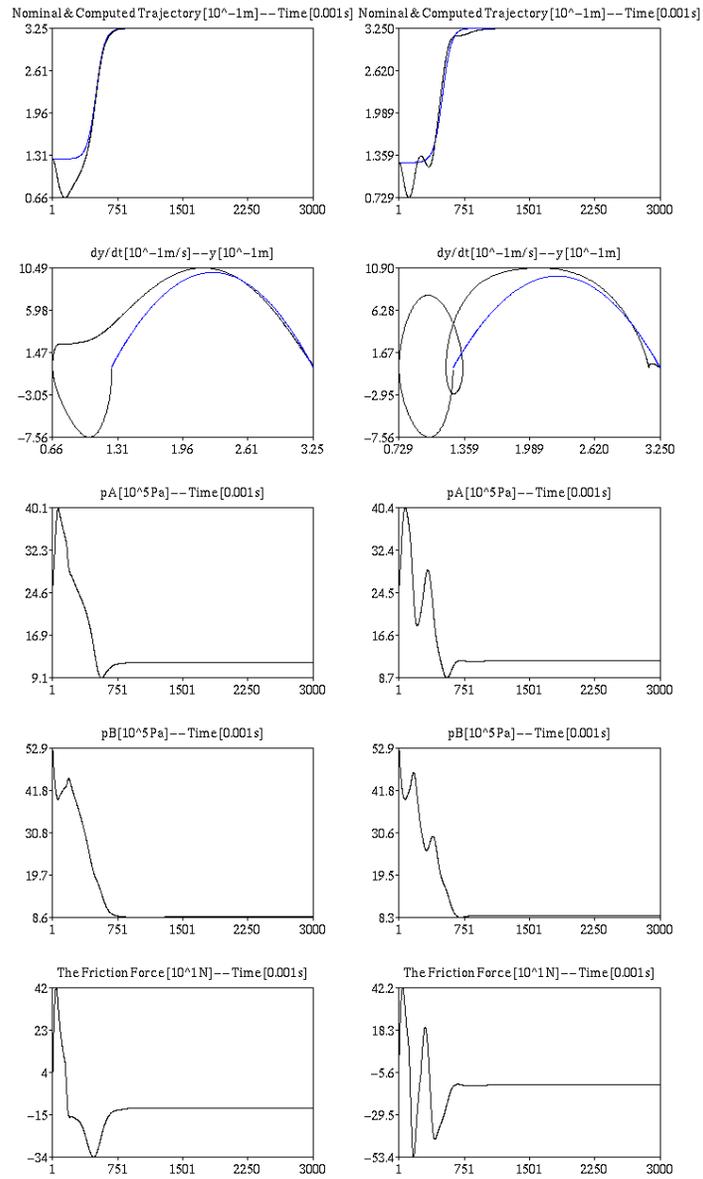


Figure 1

Trajectory tracking (1<sup>st</sup> row), phase trajectory tracking (2<sup>nd</sup> row), chamber pressures (3<sup>rd</sup> and 4<sup>th</sup> rows), and the friction force (5<sup>th</sup> row) for the fully adaptive (left column), and the fixed 3<sup>rd</sup> derivatives based control (right column) for an asymptotic nominal trajectory without periodic disturbance force component

## 4 Simulation Results

With the exception of the parameter  $E_{oil}$  all the other parameters given by Bröcker and Lemmen in [1] were used. For  $E_{oil}$  Bröcker used  $1800 \times 10^6 Pa$ , which is a huge value representing the approximate incompressibility of liquids. However, in a pipe system, due to the elasticity of the pipe walls, or due to complementary components intentionally built into the system to reduce this huge stiffness (e.g. via using hydraulic accumulators, flexible hoses) this value can be considerably smaller. In this paper  $18 \times 10^6 Pa$  was used in the simulations. Regarding the friction model's parameters we supposed to have  $\sigma_0=2500 N/m$ ,  $\sigma_1=500 Ns/m$ . The other parameters were taken from the result of identification made by Bröcker and Lemmen in [1]:  $F_C=120 N$ ,  $F_S=180 N$ ,  $F_v=175 Ns/m$ ,  $c_s=0.019 m/s$ . In each case the external disturbance force had 500 N constant component.

Fig. 1 belongs to the lack of periodic disturbance force. It convinces the reader that the application of the varying order derivation yields considerable improvement in the control, regarding both the phase trajectory and trajectory tracking. It is obvious that the *LuGre Model* with its 'hidden' internal degree of deformation results in more "treatable" description of friction than the rough formal model of sticking in which within the velocity limit both the velocity and the acceleration becomes zero until the driving force exceeds  $F_C+F_S$ . In Fig. 2 the effect of the periodic disturbance force of amplitude 200 N is observed. Since there is no possibility to directly measure the disturbance forces, its presence can be revealed only by observing the behavior of the controlled system. Due to the principle of causality this fact has to reveal itself in the small tracking error that is observed by the controller in order to compensate it. The LuGre friction considerably influences the internal variables of the adaptive control that can best be traced in the zoomed excerpts of the phase trajectories near the asymptotic end of the nominal trajectory and in the variation of the  $\|S_n - \mathbf{I}\|$  norm versus time. Fig. 3 reveals that application of the time-varying order fractional derivative results in a smoother control in which the reduced feedback generates smaller friction forces than the fixed 3<sup>rd</sup> order derivative based control.

### Conclusions

To compensate the effect of the imprecisely known system parameters and unknown external forces in a servo valve controlled differential hydraulic cylinder in this paper an adaptive control was developed. In the control varying order fractional derivatives are used to reduce the hectic behavior of the friction force in the case of the 'critical' phases of the trajectories nearby the zero velocity.

In this paper the friction was modeled in the simulations by the complex, dynamic LuGre model. Another important source of nonlinearity in such systems is of hydrodynamic origin.

The adaptive part of the controller uses a method belonging to a novel, ‘scalable’ branch of soft computing into which various Lie groups can be incorporated as the sources of uniform structures and procedures. In this case special symplectic matrices were applied.

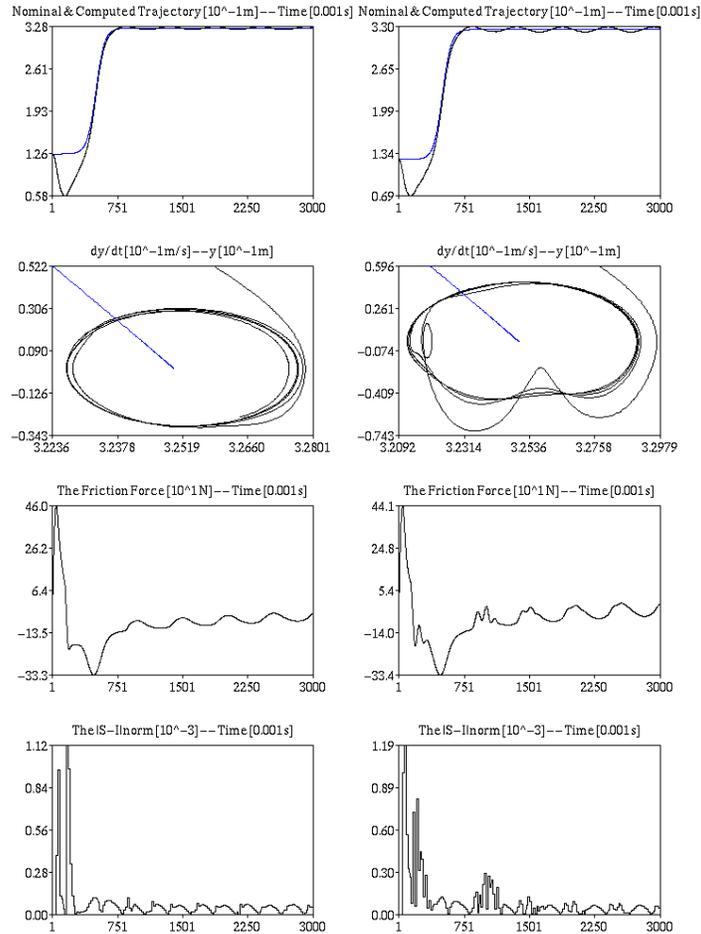


Figure 2

Trajectory tracking (1<sup>st</sup> row), critical excerpt of the phase trajectory tracking (2<sup>nd</sup> row), friction force (3<sup>rd</sup> row), and the variation adaptive matrix (4<sup>th</sup> row) for the fully adaptive (left column), and the fixed 3<sup>rd</sup> derivatives based control (right column) for an asymptotic nominal trajectory with 200 N periodic disturbance force component

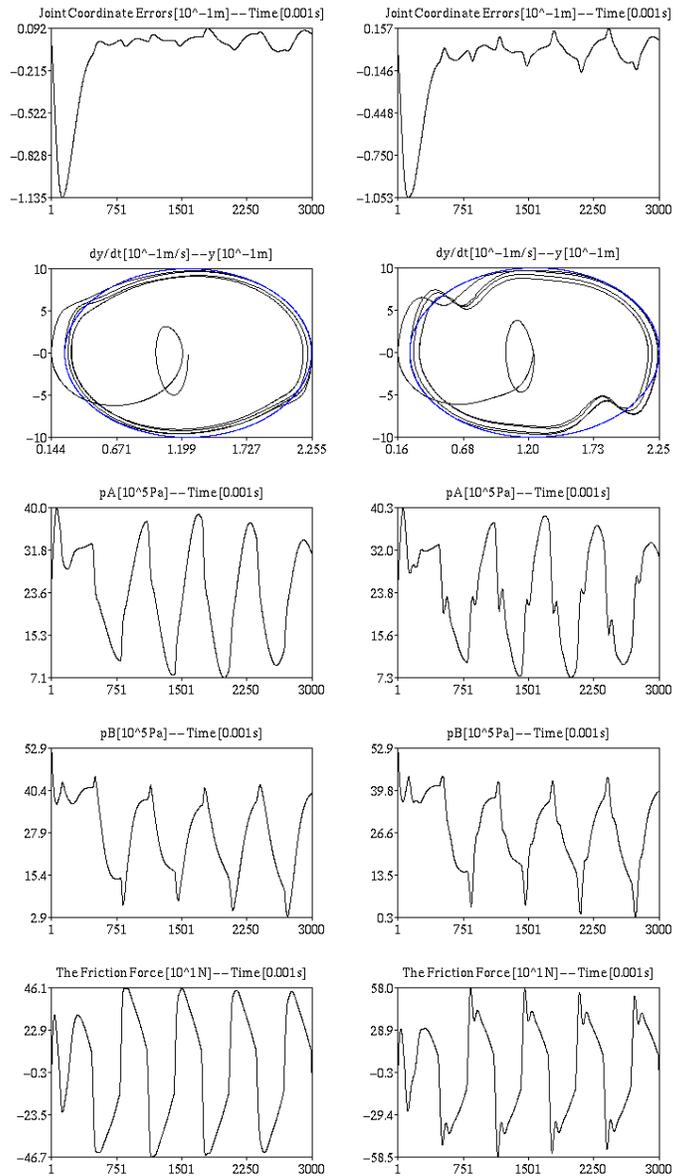


Figure 3

Trajectory tracking error (1<sup>st</sup> row), phase trajectory tracking (2<sup>nd</sup> row), chamber pressures (3<sup>rd</sup> and 4<sup>th</sup> rows), and the friction force (5<sup>th</sup> row) for the fully adaptive (left column), and the fixed 3<sup>rd</sup> derivatives based control (right column) for periodic nominal trajectory with periodic disturbance force component of amplitude 200 N

It was found that the application of the varying order derivation yields considerable improvement in the control, regarding both the phase trajectory and trajectory tracking. It is obvious that the *LuGre Model* with its 'hidden' internal degree of deformation results in more 'treatable' description of friction than the rough formal static Striebeck model of sticking in which within certain velocity limit both the velocity and the acceleration becomes zero until the driving force exceeds a force limit. This behavior of the LuGre model is very important in the low velocity part of the motion when a final fixed position has to be achieved. Furthermore, smoother control can be achieved in which the reduced feedback generates smaller friction forces than the fixed 3<sup>rd</sup> order derivative based control.

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