Similarity Measure Supported Fuzzy Failure Mode and Effect Analysis

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Abstract: Nowadays in the various engineering fields quality requirements are continuously increasing. There is also a need to develop flexible and highly adaptive systems to meet current requirements. At the same time, it is also essential to predict possible system failures and to address the arising problems appropriately. A widely used approach for predicting and preventing system failures is the Failure Mode and Effect Analysis (FMEA), which accompanies the entire development process and is able to adapt to changes in the system. The conventional method can be improved if fuzzy logic is incorporated into the evaluation. In this way the often arising subjectivity and uncertainty can be handled to ensure a more reliable result. In this paper, the author proposes a Fuzzy-FMEA (F-FMEA) based approach supported by similarity measures, for the system level. In the evaluation fuzzy arithmetic operations are applied to determine the Probability of Failure for the different failure codes. In addition to the single-expert F-FMEA system, the evaluation method that takes into account the opinions of multiple experts is also presented.

Keywords: risk assessment; expert system; Fuzzy Failure Mode and Effect Analysis; similarity measures

1 Introduction

As a consequence of the rapid development of technology, not only the opportunities in the engineering field are expanding, but at the same time the quality requirements are also increasing, while continuous availability must be ensured as well. These criteria call for a flexible, highly adaptive system that can be operated with high reliability. In order to ensure reliable operation, it is not only necessary to choose the right manufacturing method, but continuous failure-free operation, and quick identification and management of any failures that may arise are also indispensable [1], [2]. One of the most frequently used methods is the Failure Mode and Effect Analysis (FMEA), which is suitable for predicting and preventing system errors already in the planning phase, and can be continued throughout the entire life of the product or service. During the analysis, all possible events that could cause failure in the system during the process are classified and ranked. In the traditional crisp FMEA method, the level of risk can be specified with numerical values between 1-1000. However, these values are difficult to quantify since tasks of this nature are full of uncertainty and subjectivity. This problem can be addressed using the fuzzy approach, as it uses linguistic terms and can handle the subjectivity, inaccuracy and uncertainty that arise during evaluation [3].

Due to the aforementioned advantageous properties of fuzzy logic, the reliability of the model can be significantly increased. In the Fuzzy FMEA (F-FMEA), instead of numerical risk values, fuzzy sets are used in the model. In the literature several papers are available related to the F-FMEA-based failure predicting and preventing method. N. Chanamool and T. Naenna developed a fuzzy FMEA model suitable for prioritizing and evaluating possible failures in the work processes of the emergency department to choose the appropriate action and increase the confidence on hospitals [4]. G. Jin, Q. Meng and W. Feng proposed an AHP (Analytic Hierarchy Process) supported F-FMEA method to analyze the causes of failure of the logistics system. In the system the weight of the risk indicators was determined using the AHP method [5]. In the paper of X. Hu, J. Liu and Y. Wang an ontology-based F-FMEA model is introduced, in which the rating based on entropy weight and fuzzy TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) [15], [7].

The above systems can work effectively if the evaluation of possible failures has to be compiled based on the unified opinion of a single group of experts. However, there may be disagreement within the group; and in order to make the assessment more reliable, it is worth asking for the opinions of several groups of experts, which may also differ. Handling different opinions properly is a considerable challenge since a consensus has to be arrived at [8]. In the literature several different methods are presented to address this problem, such as Ordered Weighted Averaging to aggregate expert preferences [9], consistency-based algorithms using fuzzy preference relations [10], or similarity measures.

Similarity measures are widely used in risk assessment based on its advantageous properties. This approach has favourable computational requirements because it can be calculated by comparing simple features of fuzzy sets [11], [12].

In this paper, the author makes a general, flexible proposal for similarity measure-based Fuzzy Failure Mode and Effect Analysis (SF-FMEA) model to specify the probability of the potential failures (PoF) focusing on the system level. Due to the fuzzy approach FMEA components are represented by fuzzy sets taking the advantage of using linguistic terms, and the manageability of uncertainties. In the system Consequence of Failure (CoF) is also considered for each potential failure codes to determine the overall system result. The author made a proposal both for the case when the opinion of a single unified group of experts is available, and when the potentially different opinions of several different groups must be taken into account. The current PoF values were determined using fuzzy arithmetic operators, then the result was compared to the reference fuzzy sets using similarity measures to determine the current result. Furthermore, a similarity measure-based method was introduced to specify the magnitude of the consensus in the multiple-expert case to define a weight factor, which can be used in the aggregation of the expert groups' opinion.

The paper is organized as follows: In Section 2 the basic concepts related to fuzzy set theory, fuzzy operations, Fuzzy Failure Mode and Effect Analysis and similarity measures are defined. Section 3 presents the proposed similarity supported F-FMEA in two subsections: Subsection 3.1 introduces the case when the F-FMEA is prepared based on the opinion of a single expert group, while Subsection 3.2 considers the case when the opinion of multiple expert group is available, which may differ. In Section 4 a method is proposed to define the magnitude of the consensus between the different experts' opinion, and based on this value a weight factor is defined to calculate the aggregated experts's opinion. Then, in the Conclusions section the results are summarized.

2 Applied Methods

2.1 Preliminaries

This section outlines the definitions of concepts essential for the presentation of the methods.

Generalized fuzzy set: Fuzziness can be represented by a fuzzy set, which is devoted to specify the extent to which the element belongs to the set (membership degree). The fuzzy set, $A(a,b,c,d,h_A)$ is determined by a continuous mapping (membership function) from R to the closed interval [0,1]. Trapezoidal membership function is represented by (1).

$$\mu_A(x)=\begin{cases} h_A\dfrac{x-a}{b-a} & \text{if} \quad a\le x\le b \\ h_A & \text{if} \quad b\le x\le c \\ h_A\dfrac{d-x}{d-c} & \text{if} \quad c\le x\le d \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $a \neq b, c \neq d$ and h_A is the maximum value of the set, $h_A \in]0,1]$ [13].

Normality of a fuzzy set: The normality of the fuzzy set A is basically determined by its highest value (h_A). In most cases normal sets, i.e. $h_A = 1$, are used. However, in the case of generalized fuzzy sets lower value is also allowed, i.e. $0 < h_A \le 1$ [13].

Defuzzification: Defuzzification is a process when a fuzzy set (e.g. system result) should be represented by a suitable crisp value. The most commonly used method is the Centre of Gravity (CoG), which can assign a crisp value to any shape set properly. However, the greatest disadvantage of this approach is the high computational requirement. To handle this drawback the Simplified Centre of Gravity (SCoG) method is used in this study. The basis of this approach is the center curve of the trapezoidal fuzzy set [11], [14] which can be calculated for trapezoidal sets, $A(a, b, c, d, h_A)$ according to (2), (3).

$$y_{SCoG_A} = \frac{h_A\left(\frac{c-b}{d-a}+2\right)}{6} \tag{2}$$

$$x_{SCoG_A} = \frac{y_{SCoG_A}(c+b)+(d+a)\left(h_A - y_{SCoG_A}\right)}{2h_A} \tag{3}$$

Fuzzy arithmetic operations: In order to be able to perform operations with generalized fuzzy numbers, arithmetic operations should be defined. In this study, Chen's operators are used for the above defined fuzzy sets $\left(A_1(a_1, b_1, c_1, d_1, h_{A_1});\ A_2(a_2, b_2, c_2, d_2, h_{A_2})\right)$ as follows [15]:

Addition: $(A_1 \oplus A_2) = \left(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, min(h_{A_1}, h_{A_2})\right)$ (4)

Subtraction: $(A_1 \ominus A_2) = \left(a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2, min(h_{A_1}, h_{A_2})\right)$ (5)

Multiplication: $(A_1 \otimes A_2) = \left(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2, min(h_{A_1}, h_{A_2})\right)$ (6)

Division: $(A_1 \oslash A_2) = \left(\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}, \frac{d_1}{d_2}, min(h_{A_1}, h_{A_2})\right)$ (7)

2.2 Fuzzy Failure Mode and Effect Analysis

Failure Mode and Effect Analysis (FMEA) is an effective technique for predicting and preventing system failures. It is a commonly used approach in manufacturing systems, mainly in those that produce safety-critical products and contain advanced electronic and mechanical equipment based on system analysis [16]. The essence of the method is to qualify and prioritize the random and natural events occurring in the system during the process, which can cause damage. Each failure mode is characterized by three metrics: Consequence of Failure (*CoF*), Probability of Failure (*PoF*) and Detectability of Failure (*DoF*). These three aspects are often referred to in the literature as Severity (here *CoF*), Occurrence (here *PoF*), and Detectability (here *DoF*). The crisp FMEA method is based on a numerical scale, ranging from 1 to 10, where 1 is the lowest risk and 10 is the highest. Taking into account these three characteristics together, Risk Priority Number (*RPN*) has to be calculated, using (8) to be able to rank the particular risk scenarios.

$$\mathrm{RPN_i} = \mathrm{CoF_i} \cdot \mathrm{PoF_i} \cdot \mathrm{DoF_i} \tag{8}$$

where $i \in [1, n]$, n is the number of the different failure codes.

These kinds of tasks are full of uncertainties and subjectivity. The fuzzy approach is a good solution for this problem because it is able to handle subjectivity, imprecision and uncertainty in the evaluation. In this way the reliability of the model can be significantly increased. In order to fuzzify the process *CoF*, *PoF* and *DoF* should be reprezented by fuzzy sets instead of crisp numbers. These sets have to be a partition of the [0,10] interval. In this study, [0,1] interval is used proportionally due to later calculations. Fuzzy sets representing *CoF*, *PoF* and *DoF* values are illustrated in Fig. 1 [17].

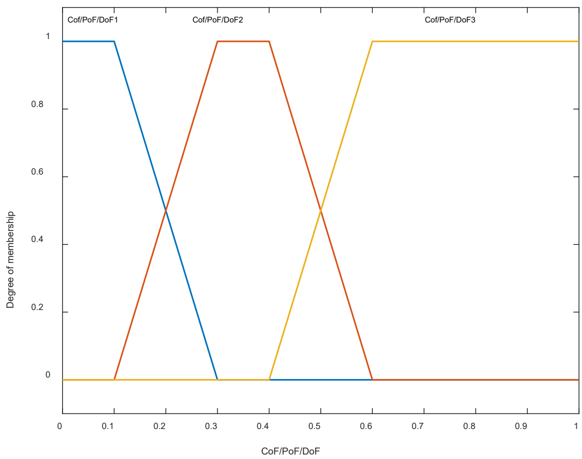

Figure 1
CoF, PoF and DoF fuzzy sets (Linguistic terms for CoF/PoF/DoF : CoF/PoF/DoF1= Low/Improbable/EasilyDetectable, CoF/PoF/DoF2= Medium/Occasional/Detectable, CoF/PoF/DoF3= High/Probable/HardlyDetectable, respectively)

In the Fuzzy FMEA (F-FMEA) the *RPN* value is determined by a fuzzy inference system, where the evaluation is based on a rule base [4]. In this study, the Mamdani-type inference is used, i.e. the output domain is also covered by fuzzy sets (see Fig. 2).

The input of the F-FMEA can be a crisp value or even a fuzzy number. However, fuzzy set-represented expert knowledge is more informative. Consequently, in this study, the fuzzy number type opinions are considered. Similarly to the crisp FMEA, in its fuzzy version each failure code has to be evaluated using the above method.

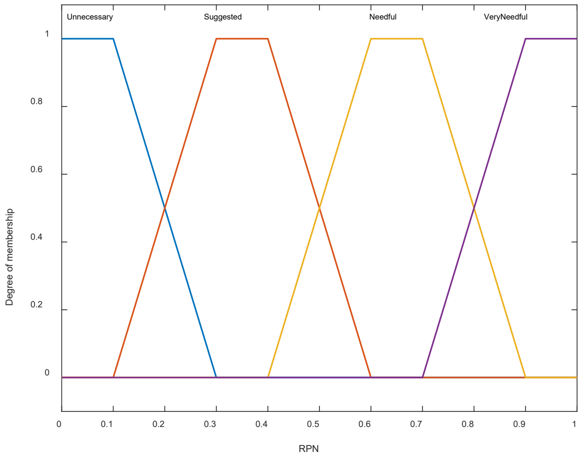

Figure 2
RPN fuzzy sets representing the necessity of action

2.3 Similarity Measures

Similarity measures are used to compare fuzzy sets and numbers calculating the degree of similarity, $0 < S(A_1, A_2) \leq 1$, where A_1, A_2 are fuzzy sets or numbers. If the similarity value is 1, the fuzzy sets are the same. The lower its value the greater the difference between the sets. Similarity is determined based on the characteristics of fuzzy sets, there are many different approaches. In this study, the fuzzy set parameters and the defuzzified value are used for the comparison, calculating the SCoG value for each fuzzy set by (2), (3). Similarity calculation can be performed using (9), (10), (11).

$$S(A_1, A_2) = c\left(1 - \frac{|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|}{4}\right)\Delta x_{SCoG} M \tag{9}$$

where

$$\Delta x_{SCoG} = 1 - \left| x_{SCoG_{A_1}} - x_{SCoG_{A_2}} \right| \tag{10}$$

$$M = \frac{\min\left(y_{SCoG_{A_1}}, y_{SCoG_{A_2}}\right)}{\max\left(y_{SCoG_{A_1}}, y_{SCoG_{A_2}}\right)} \tag{11}$$

c is a constant to specify the direction of the deviation, if needed. If the direction is not relevant, or $x_{SCoG_{A_1}} \geq x_{SCoG_{A_2}}$ then $c = 1$. If $x_{SCoG_{A_1}} < x_{SCoG_{A_2}}$ then $c = -1$.

3 Fuzzy Failure Mode and Effect Analysis using Simiarity Measures

In the F-FMEA method the potential failures of the system can be represented by three main components, such as the Probability of Failure (*PoF*), the Consequence of Failure (*CoF*), and the Detectability of Failure (*DoF*). In the fuzzy approach these components are characterized by fuzzy sets taking the advantage of the use of linguistic terms and the manageability of uncertainties. In this study, the focus is on the *PoF* value. The main goal is to determine its overall value taking into account all failure codes determined by the experts. In this paper, the failure codes are not specified, as this is a general suggestion that can be flexibly applied to different specific systems, and failure codes.

3.1 Single Expert Case

In the F-FMEA process, the fuzzy reference sets shown in Figure 1 are used both during the expert classification of individual error codes, and the overall system output is compared with them.

Let the failure codes be $C_1, C_2, \dots, C_n$, where *n* is the number of the potential failures, and each C_i is characterized by its corresponding PoF_i and CoF_i. The overall PoF of the system is determined by the fuzzy weighted average calculated using fuzzy arithmetic operations (see 2.1) as follows:

$$\mathrm{PoF_O} = \frac{\sum_{i=1}^{n} \mathrm{CoF_i} \otimes \mathrm{PoF_i}}{\sum_{i=1}^{n} \mathrm{CoF_i}} \tag{12}$$

Based on the calculated *PoF* parameters, which represent a normal fuzzy set, the probability of an error occurring in the system can be determined using similarity measures. The overall PoF set (PoF_O) and reference PoF sets (Fig. 1) should be compared by (9), (10), (11). Based on the highest similarity value, it can be determined which of the reference sets the overall *PoF* is closest to.

Let the number of the failure codes be 5, for which the expert opinion is defined according to Table 1.

Using (12), the overall *PoF* value is as follows:

$\mathrm{PoF_O} = (0.21, 0.35, 0.58, 0.67)$ as illustrated in Fig 3.

Table 1

Expert's opinion for the different failure codes

Failure code	CoF	$CoF_i(a_1, b_1, c_1, d_1)$	PoF	$PoF_i(a_1, b_1, c_1, d_1)$
Failure1	Low	(0,0,0.1,0.3)	Improbable	(0,0,0.1,0.3)
Failure2	Medium	(0.1,0.3,0.4,0.6)	Probable	(0.4,0.6,1,1)
Failure3	High	(0.4,0.6,1,1)	Probable	(0.4,0.6,1,1)
Failure4	High	(0.4,0.6,1,1)	Improbable	(0,0,0.1,0.3)
Failure5	Medium	(0.1,0.3,0.4,0.6)	Occasional	(0.1,0.3,0.4,0.6)

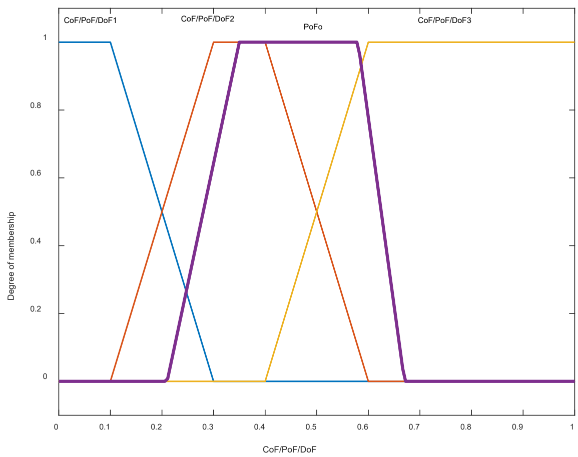

Figure 3

Comparison of the overall PoF set to the reference sets (CoF/PoF/DoF1, CoF/PoF/DoF2, CoF/PoF/DoF3 represent the reference sets, while PoFo is the overall PoF set)

After the PoF_O value is available, one has to compare it to the reference fuzzy sets (see Fig. 1) to obtain the final result. Similarity values for all reference sets are listed in Table 2. The highest value determines which linguistic variable can be assigned to the *PoF* value of the overall system. It can be seen that the highest value is 0.714, and the associated fuzzy set is PoF_2 representing the linguistic term *Occasional*. This result means that intervention may be necessary to avoid the occurence of a potential failure.

Table 2

Similarity values of the overall PoF and reference sets

PoF_i	$S(PoF_i, PoF_O)$
PoF_1 (Improbable)	0.402
PoF_2 (Occasional)	0.714
PoF_3 (Probable)	0.466

3.2 Aggregated Experts' Opinion-based Evaluation

In order to compile an effective FMEA, one should consider the opinions of several experts. However, these opinions may differ, requiring great care to be handled appropriately. In this section, the author proposes the multiexpert version of the similarity measures supported FMEA to address the above problem.

This method is an extension of the similarity measures supported evaluation process (see 3.1). In the above described case only one expert's opinion is available, therefore, normal fuzzy sets can be used effectively. However, in the multiexpert version, the opinions of several experts, which may differ, should all be taken into account. For this reason, these opinions have to be weighted based on the degree of confidence of the experts using subnormal fuzzy sets. The height of the generalized fuzzy set is used to represent the degree of confidence (DoC) of each expert. First, the problem is reduced by averaging the different opinions for each failure code by (13), (14) determining the average *PoF* value $\left(PoF_{avg_i}\right)$.

$$\mathrm{PoF}_{\mathrm{avg}_i} = \frac{\sum_{j=1}^{m} \mathrm{PoF}_{ij}}{m} \tag{13}$$

$$\mathrm{PoF}_{\mathrm{avg}_i} = \frac{\sum_{j=1}^{m} a_{ij}}{m}, \frac{\sum_{j=1}^{m} b_{ij}}{m}, \frac{\sum_{j=1}^{m} c_{ij}}{m}, \frac{\sum_{j=1}^{m} d_{ij}}{m}, \frac{\sum_{j=1}^{m} h_{ij}}{m} \tag{14}$$

$$\mathrm{PoF}_{\mathrm{avg}_i} = a_{\mathrm{avg}_i}, b_{\mathrm{avg}_i}, c_{\mathrm{avg}_i}, d_{\mathrm{avg}_i}, h_{A_{\mathrm{avg}_i}} \tag{15}$$

where $j \in [1, m]$, m is the number of the expert teams, $i \in [1, n]$, n is the number of the different failure codes, $a_{ij}, b_{ij}, c_{ij}, d_{ij}, h_{ij}$ are the generalized fuzzy set parameters for failure code i, and expert j, while $a_{avg_i}, b_{avg_i}, c_{avg_i}, d_{avg_i}, h_{A_{avg_i}}$ represent the average fuzzy set parameters for failure code i.

Following the aggregation, the process is the same as in the original (single expert) case, but instead of the opinion of the single expert, the above calculated average *PoF* value $\left(PoF_{avg_i}\right)$ is used. The next step is the overall *PoF* value calculation by (16), then the obtained generalized fuzzy number $\left(PoF_{avg_O}\right)$ compared to the reference fuzzy sets specified in Fig 1. Comparison is performed by similarity measure using (9), (10), (11) and the reference set for which the largest value obtained represents the system result.

$$\mathrm{PoF}_{\mathrm{avg}_O} = \frac{\sum_{i=1}^{n} \mathrm{CoF}_i \otimes \mathrm{PoF}_{\mathrm{avg}_i}}{\sum_{i=1}^{n} \mathrm{CoF}_i} \tag{16}$$

where $i \in [1, n]$, n is the number of the failure codes.

Let the number of the failure codes be 5, and the number of the different expert groups be 3. The opinion of the groups are presented in Table 3, where the Degree of Confidence (DoC) of the groups are represented by the height of the fuzzy sets.

Table 3

Expert groups' opinion for the different failure codes

Failure code	Group1 (DoC=0.9)	Group2 (DoC=0.7)	Group3 (DoC=0.8)
Failure1	Improbable	Occasional	Improbable
Failure2	Probable	Occasional	Occasional
Failure3	Probable	Occasional	Occasional
Failure4	Improbable	Improbable	Occasional
Failure5	Occasional	Probable	Probable

First, average *PoF* value should be calculated by taking into account the opinion of all expert groups using (13), (14), (15). These average values are summarized in Table 4.

Table 4

Average PoF and CoF values for each failure code

Failure code	$\mathrm{PoF}_{\mathrm{avg}_i}$	CoF_i
Failure1	(0.033,0.1,0.2,0.4)	(0,0,0.1,0.3)
Failure2	(0.2,0.4,0.6,0.733)	(0.1,0.3,0.4,0.6)
Failure3	(0.2,0.4,0.6,0.733)	(0.4,0.6,1,1)
Failure4	(0.033,0.1,0.2,0.4)	(0.4,0.6,1,1)
Failure5	(0.3,0.5,0.8,0.867)	(0.1,0.3,0.4,0.6)

After the average values are available, the PoF value can be calculated for the overall system in the same way as in the single expert case, and the overall PoF set is as follows:

$$\mathrm{PoF}_{\mathrm{avg}_O} = (0.143, 0.317, 0.476, 0.632)$$

The final step of the process is to compare the overall PoF value to the reference fuzzy sets (see Fig. 1). The degree of similarities are presented in Table 5.

Table 5
Similarity values of the overall PoF and reference sets

PoF_i	$S(PoF_i, PoF_O)$
PoF_1 (Improbable)	0.406
PoF_2 (Occasional)	0.777
PoF_3 (Probable)	0.292

The highest value determines which linguistic variable can be assigned to the PoF value of the overall system. It can be seen that the highest value is 0.777, and the associated fuzzy set is PoF_2, representing the linguistic term *Occasional*. This result means that intervention may be necessary to avoid the occurence of a potential failure.

4 Consensus-based Similarity Supported FMEA Model

In this section a comparison method is introduced, whose main purpose is to represent the magnitude of the consensus between the different experts' opinion. Then, based on the obtained value a weight factor is defined, by which the aggregated experts's opinion can be calculated.

In this study, the comparison is performed taking into account the *PoF* value based on the experts' opinion, represented by fuzzy sets. During the evaluation subnormal fuzzy sets $A(a, b, c, d, h_A)$ are applied, where the height of the set (h_A) represents the degree of confidence associated with each expert.

To determine the degree of consensus, one should perform the following process for each failure code:

1. Fuzzy set $A(a, b, c, d, h_A)$ creation based on the experts' opinion
2. SCoG (x_{SCoG}, y_{SCoG}) value is calculated for each fuzzy set by (2), (3)
3. Similarity calculation to compare the sets by (9), (10), (11)

The result of this process is the magnitude of the consensus between the different experts. The greater the obtained values the higher the consensus. Its maximum value is 1, which means that the different experts completely agree on the specific error code. Based on the magnitude of the consensus a weight factor can be specified to use when the experts' opinion are aggregated. Fuzzy sets with identical or nearly identical parameters can be represented by a single set. Then, the number of these kinds of sets is used to calculate the weight factor (w_j) of this single set. This weight factor ensures the work with normal fuzzy sets, i.e., instead of the height of the set, a weight factor is used. In this case, the average *PoF* value can be calculated as follows:

$$PoF_{wavg_i} = \frac{\sum_{j=1}^{l} w_j PoF_{ij}}{\sum_{j=1}^{l} w_j} \tag{17}$$

$$PoF_{wavg_i} = \frac{\sum_{j=1}^{m} w_j a_{ij}}{\sum_{j=1}^{l} w_j}, \frac{\sum_{j=1}^{m} w_j b_{ij}}{\sum_{j=1}^{l} w_j}, \frac{\sum_{j=1}^{m} w_j c_{ij}}{\sum_{j=1}^{l} w_j}, \frac{\sum_{j=1}^{m} w_j d_{ij}}{\sum_{j=1}^{l} w_j} \tag{18}$$

$$PoF_{wavg_i} = a_{wavg_i}, b_{wavg_i}, c_{wavg_i}, d_{wavg_i} \tag{19}$$

where fuzzy sets are represented by their basic parameters $A(a,b,c,d)$, $j \in [1,l]$, l is the number of the different fuzzy sets, w_j is the weigth factor of fuzzy set j, $i \in [1,n]$, n is the number of the different failure codes, $a_{ij}, b_{ij}, c_{ij}, d_{ij}$ are the normal fuzzy set parameters for failure code i, and fuzzy set j, while $a_{wavg_i}, b_{wavg_i}, c_{wavg_i}, d_{wavg_i}$ represent the average fuzzy set parameters for failure code i.

Let the number of the failure codes be 5, and the number of the different expert groups be 3. The opinion of the groups are presented in Table 3, but normal fuzzy sets are used. First, the similarity degree is calculated for expert groups in pairs for each failure code separately. These values represent the magnitude of the consensus. Based on these results fuzzy sets with identical or nearly identical parameters can be represented by a single set, whose weight is determined accordingly.

Table 6
Similarity values for each failure code

Failure code	S(Group1,Group2)	S(Group1,Group3)	S(Group2,Group3)
Failure1	0.538	1.000	0.538
Failure2	0.300	0.300	1.000
Failure3	0.300	0.300	1.000
Failure4	1.000	0.538	0.538
Failure5	0.300	0.300	1.000

Based on Table 6 it can be seen which fuzzy sets are identical (similarity values are 1). These sets are represented by a single set and their weight is doubled. The resulting sets are then averaged using (17), (18), (19) as illustrated in Table 7.

Table 7

Weighted average of PoF and CoF values for each failure code

Failure code	PoF_{avg_i}	CoF_i
Failure1	(0.02,0.06,0.16,0.36)	(0,0,0.1,0.3)
Failure2	(0.16,0.36,0.52,0.68)	(0.1,0.3,0.4,0.6)
Failure3	(0.16,0.36,0.52,0.68)	(0.4,0.6,1,1)
Failure4	(0.02,0.06,0.16,0.36)	(0.4,0.6,1,1)
Failure5	(0.34,0.54,0.88,0.92)	(0.1,0.3,0.4,0.6)

After the average values are available, the PoF value can be calculated for the overall system in the same way as in the single expert and DoC-based multiexpert case. The overall PoF set is as follows:

$$PoF_{wavg_O} = \left(0.122, 0.29, 0.433, 0.602\right)$$

The final step of the process is to compare the overall PoF value to the reference fuzzy sets (see Fig. 1). The degree of similarities are presented in Table 6.

Table 8

Similarity values of the overall PoF and reference sets

PoF_i	$S(PoF_i, PoF_O)$
PoF_1 (Improbable)	0.382
PoF_2 (Occasional)	0.720
PoF_3 (Probable)	0.446

The highest value determines which linguistic variable can be assigned to the PoF value of the overall system. It can be seen that the highest value is 0.720, and the associated fuzzy set is PoF_2, representing the linguistic term *Occasional*. This result means that intervention may be necessary to avoid the occurence of a potential failure.

Comparing the results of the DoC-based and consensus-based approach (see Table 9), it is clear that the highest similarity can be seen with reference set 2 in both cases. However, for the other two reference sets, the similarity is reversed. In the consensus-based model, the result is shifted to the PoF_3, which means that it makes the occurence of a potential failure in the system more likely.

Table 9

Comparison of the DoC-based and consensus-based models

PoF_i	$S(PoF_i, PoF_O)$	$S(PoF_i, PoF_O)$
PoF_1 (Improbable)	0.406	0.382
PoF_2 (Occasional)	0.777	0.720
PoF_3 (Probable)	0.292	0.446

Conclusions

In engineering systems, it is not only necessary to apply the technologically appropriate method, but also to continuously avoid any unwanted events. Failure Mode and Effect Analysis is one of the most commonly used approaches suitable for the quick identification and management of potential failures in the system. The extension of this method with fuzzy logic (F-FMEA) makes it possible to handle uncertainties, subjectivity and imprecision in the evaluation.

In this paper a similarity measure-based F-FMEA model was proposed for determining the overall Probability of Failure in the system. Similarity measures are very popular in risk assessment applications because of their favourable computational properties. In this study, different potential failures were considered characterized by their PoF and CoF values. During the evaluation fuzzy arithmetic operators were used to determine the overall system result. Then, the results were interpreted based on the comparison with the reference fuzzy sets. The basic method takes into account a single expert's opinion. However, in order to make the results of the system more reliable, the opinions of several experts must be taken into account. In this case, the greatest challenge is that the opinion of the expert groups can often be different. For this reason. author also proposed a multiexpert vesion of the similarity supported F-FMEA to address the above problem. In this DoC-based model the Degree of Confidence for each expert groups was considered, which is represented by the height of the generalized fuzzy sets. Furthermore, the magnitude of the consensus between the different expert groups was also calculated using similarity measures. Then, based on the obtained result, a weight factor was defined, which was used in the overall PoF value calculation.

The methods were illustrated by numerical examples and the results of the DoC-, and consensus-based methods were compared. The comparison resulted in the same linguistic term as the system result. However, for the other two reference sets, the similarity was reversed.

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