

Type-2 Fuzzy Sets and SSAD as a Possible Application

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Abstract: In the paper a short review of basic type-2 terms is given. One of the possible type-2 Fuzzy Logic System applications is represented for signal processing problems, because type-2 FLSs can handle second level of uncertainties of the stochastic error measurements using Stochastic adding A/D conversion.

Keywords: type-2 fuzzy sets, distance based operators

1 Introduction

Since the beginning of the fuzzy world, there are applications of type-1 fuzzy systems (FS) in which fuzzy system are used to approximate random data or to model an environment that is changing in an unknown way with time [1]. L. A. Zadeh introduced type-2 and higher-types FS in 1975 [6], to eliminate the paradox of type-1 fuzzy systems which can be formulated as the problem that the membership grades are themselves precise real numbers. It is not a serious problem for many applications, but in the cases when the data generating system is known to be time-varying but the mathematical description of the time-variability is unknown, or when the measurement noise is non-stationary and the mathematical description of the non-stationarity is unknown, furthermore when the features in a pattern recognition application have statistical attributes that are non-stationary and the mathematical description of the non-stationarity is

unknown, or knowledge is gathered from a group of experts using questionnaires that involve uncertain words linguistic terms are used and have a non-measurable domain, the results of type-1 fuzziness are imprecise boundaries of FS-s [5].

The solution for this problem can be type-2 fuzziness, where fuzzy sets have grades of membership that are themselves fuzzy [1], [2]. At each value of the primary variable x on the universe X , the membership is a function, and not just a point value (characteristic value). It is the second level, or secondary membership function, whose domain is the primary membership value set. The secondary membership function is a function $MF2 : [0,1] \rightarrow [0,1]$. It can be concluded that $MF2$ gives a type-2 fuzzy set which is three-dimensional, and the third dimension offers a certain degree of freedom for handling uncertainties.

In [1] Mendel defines and differentiates two types of uncertainties, random and linguistic. The first one is characteristic for example in statistical signal processing, and the linguistic uncertainties characteristic have in word-information based imprecision systems.

Operations on type-2 fuzzy sets are extended based on type-1 union, intersection, and complementation and usually apply t-norms and conorms.

In the process of T2 fuzzy logic inference the uncertainties of fuzzy membership are included in the calculation. Using the maximum and minimum operators for interval type-2 fuzzy systems (IT2 FS) the description of the Mamdani type T2 inference and defuzzification is given in [2].

In the paper a short review of basic type-2 terms is given. A possible application of this method is related to signal processing [7]. The developed measure method SAADK in [3], stochastic adding A/D conversion has several advantages compared to standard digital instrumentation. Its main advantages are extremely simple hardware and, consequently, simple implementation of parallel measurements, and on the other hand, possibility to trade speeds for accuracy. This instrument can be fast and less accurate, or slow and very accurate, depending on the choice of frequency of reading its output, therefore the membership on the universe X is time-dependant and with random uncertainties, it means suitable for representing it as T2 FS. The hardware realization of the problem is based only on the min and max operators, therefore distance based operators, which are also defined using min and max, making them applicable in the process of realization. Furthermore, it is an opportunity to use simply hardware realization of the operators and approximate reasoning process in the control problems based on the SAADK method and distance based operators [4].

2 Type-2 Fuzzy Sets

A type-1 fuzzy set (T1 FS) has a grade of membership that is crisp, whereas a type-2 fuzzy set (T2 FS) has a grade of membership that is fuzzy, so T2 FS are ‘fuzzy-fuzzy’ sets.

One way of representing the fuzzy membership of fuzzy sets is to use the footprint of uncertainty (FOU), which is a 2-D representation, with the uncertainty about the left end point of the left side of the membership function, and with the uncertainty about the right end point of the right side of the membership function. Let $x \in X$ from the universe of basic variable of interest. Let $MF(x)$ be the T1 fuzzy membership function of the fuzzy linguistic variable or other fuzzy proposition. The functions $UMF(x)$ and $LMF(x)$ are functions of the left-end and right-end point uncertainty (Fig. 1). In a fix point x' of the universe X it is possible to define so-called vertical slices of the uncertainty, describing it for different possibilities of the $MF_i(x)$ functions ($i=1,2,..N$), included in the shading of FOU.

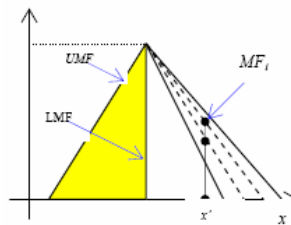


Figure 1
FOU type T2 FS

In this case, for example if we have a Gaussian primary membership function (MF), very often the uniform shading over the entire FOU means uniform weighting or uniform possibilities. T2 FS with FOU representation and uniform possibilities on FOU is called the interval type-2 FS (IT2 FS).

The second way is to use 3D representation, where in the domain xOy the F1 FS $A(x)$ is represented, and in the third dimension for every crisp membership value $A(x)$ of the basic variable x a value of possibility (or uncertainty) is given as the function $MF(x,A(x)) = \mu(x, A(x))$. It is the embedded 3D T2 FS (Fig. 2). In Fig. 2 the value $\mu(x, A(x))$ is a random value from the interval $[0,1]$.

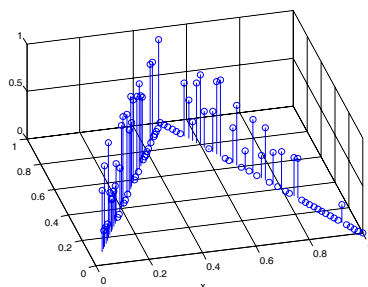


Figure 2
FOU type T2 FS

3 Stochastic Additive Analog to Digital Converter

In classical measurement in the whole measurement range absolute error is, more or less, is the same. This is characteristic of uniform quantizer, i.e. analog to digital converter. On the other hand, in case of control we know that all measurement subintervals are not of equal signification. This means that equal precision is not necessary for control applications. In less significant subintervals it is sufficient to confirm that the system is in it. No more information is needed. Fuzzy systems serve as good mathematical models for this simple situation.

An interesting question arises: does the fuzzyfication belong to domain of measurements and metrology? The answer can be found in several definitions.

Metrology is the science of measurement. Metrology includes all aspects both theoretical and practical with reference to measurements, whatever their uncertainty, and in whatever fields of science or technology they occur. Measurement is set of operations having the object of determining a value of a quantity. Measurable quantity is an attribute of a phenomenon, body or substance that may be distinguished qualitatively and determined quantitatively. Analog to digital conversion is the conversion of an analog quantity in its digital counterpart.

The possible conclusion, from the above definitions, is that fuzzyfication can be accepted as a kind of measurement, since performing fuzzyfication we confirm membership functions, i.e. that fuzzyfied quantity has characteristics of defined fuzzy sets, depending on the moment- the time variable.

Human beings are not capable to process to many information, so they process one information at a moment, possible, most significant in that moment. The process of choosing this most significant information is a kind of control process. Human have capability to do this.

Adaptive measurement system put this human characteristic in the area of automatic control systems. Let us imagine that we have system with many inputs, one multiplexer and one system for processing inputs. On input we have information with low resolution, defined with some synchronizing pulse, but using longer processing time we get information with higher resolution. If processing system works longer on specific input the information is more detailed and more reliable, in the opposite case information is rough and less reliable.

Processing cycle for all inputs lasts the same time, in every case, but work on specific inputs can be variable, depend on system state. So, there is a need for instrument capable to give rough and fast measurement information in the shorter time or precise and reliable measurement information in longer time interval. One of such instruments is stochastic additive A/D converter (SSAD).

The developed measure method, stochastic adding A/D conversion has several advantages compared to standard digital instrumentation [3]. Its main advantages are:

- extremely simple hardware and, consequently, simple implementation of parallel measurements, and
- possibility to trade speed for accuracy.

This instrument can be either fast and less accurate, or slow and very accurate. The choice can be made specifying the frequency of reading its output. This is a kind of adaptation which master processor performs. In the case of multi-channel measurements, adaptation can be performed on each channel independently.

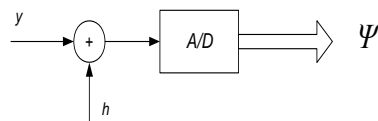


Figure 3

The most simple outline of the instrument

In the Fig. 3 a schematic of the instrument is shown. Dithering signal h is random, uniform and satisfies Widrow's conditions

$$0 \leq |h| \leq \frac{a}{2} \quad (1)$$

$$p(h) = \frac{1}{a}, \quad (2)$$

where a is a quantum of the uniform quantiser, and $p(h)$ is the corresponding probability density function of h .

3.1 Theory of Operation - DC Inputs

Let us observe the output of AD converter Ψ . Let $y = \text{const} = na + |\Delta a|$ be the corresponding input voltage located between quantum level na and $(n+1)a$, at the distance $|\Delta a| \leq a/2$ from the closest quantum level na shown in Figure 4.

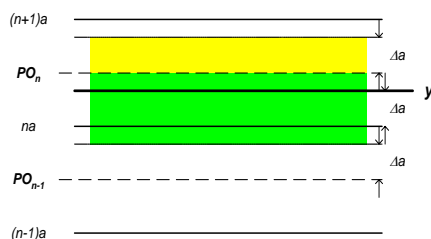


Figure 4

The situation for $y = \text{const}$

For the situation depicted in Fig. 4 the quantized level of $y+h$ is $\Psi \in \{na, (n+1)a\}$. The expectation $\bar{\Psi}$ is given by

$$\begin{aligned} \bar{\Psi} &= \Psi_1 \cdot p_1 + \Psi_2 \cdot p_2 = (n+1) \cdot a \cdot \frac{|\Delta a|}{a} + na \cdot \frac{(a-|\Delta a|)}{a} = \\ &= n \cdot |\Delta a| + |\Delta a| + na - n \cdot |\Delta a| = na + |\Delta a| = y \\ \bar{\Psi} &= y \end{aligned} \quad (3)$$

The corresponding variance is:

$$\begin{aligned} \overline{e^2} &= \sigma_{\Psi}^2 = (\Psi_1 - \bar{\Psi})^2 \cdot p_1 + (\Psi_2 - \bar{\Psi})^2 \cdot p_2 = \\ &= (a - |\Delta a|)^2 \cdot \frac{|\Delta a|}{a} + |\Delta a|^2 \cdot \frac{(a - |\Delta a|)}{a} = \\ \sigma_{\Psi}^2 &= (a - |\Delta a|) \cdot \left((a - |\Delta a|) \cdot \frac{|\Delta a|}{a} + \frac{|\Delta a|^2}{a} \right) = \\ &= (a - |\Delta a|) \cdot (|\Delta a|) \\ \sigma_{\Psi}^2 &= (a - |\Delta a|) \cdot (|\Delta a|) \end{aligned} \quad (4)$$

An interesting question is: what is the error if we have finite number N of dithered samples? The answer gives theory of samples and central limit theorem. Suppose that we have next set of samples: $\Psi_1, \Psi_2, \dots, \Psi_n$.

Then measurement quantity is

$$\bar{\Psi} = \frac{1}{N} \sum_{i=1}^N \Psi_i \approx const \quad (5)$$

Central limit theorem gives next result

$$\sigma_{\bar{\Psi}}^2 = \frac{\sigma_{\Psi}^2}{N}. \quad (6)$$

The equations (4), (5) and (6) completely define the situation if we have a finite number N of dithered samples.

The shape of $\sigma_{\bar{\Psi}}^2$ as function of Δa , for the case of $y=const$, is given in Fig. 4.

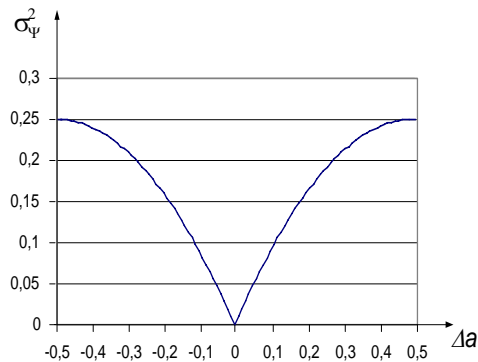


Figure 5

The diagram of $\sigma_{\bar{\Psi}}^2$ (Δa) for $y=const$ and $a=1$

From the Fig. 5 is obvious that the maximum value of error is in the case of $y = c = \frac{a}{2} + na$, i.e. when measurement voltage has value of threshold voltage, but the minimum value of error is when $y = c = na$, i.e. when measurement voltage has value of quantum level.

3.2 Function-based Measurements and the Numeric Processing of the Measurements

Stochastic additive A/D converter basically works with equidistant comparators, but this is not obligatory. If we make A/D converter with non-equidistant threshold levels, then we have stochastic additive analog to fuzzy converter (SAAFC).

Defining the membership functions on the output of the stochastic additive analog to fuzzy converter the measurement range is divided in to n intervals. There is one deciding threshold in every interval. $(n + 1)$ sets are defined by n deciding thresholds. The first set is approximately the minimum value of the measurement range, $(n+1)^{st}$ set is approximately the maximum value of the measurement range. Between these two sets there are $(n-1)$ sets 'approximately A_j ' where x_{A_j} represents the middle of the two deciding thresholds: PO_j and PO_{j-1} where $j=2, \dots, n$. The deciding thresholds are marked from PO_1 to PO_n . There exists the following limitation: $PO_1 > PO_{j-1}$ where $j = 2, \dots, n$.

In general case, the membership of the defined sets can be described in the following way:

$$A_j(x) = \{x_i | x_i \geq PO_{j-1} \wedge x_i < PO_j\} \quad \text{where } j = 2, \dots, n \quad (7)$$

For the first and the last set:

$$A_1(x) = \{x_i | x_i < PO_1\} \quad (8)$$

$$A_{n+1}(x) = \{x_i | x_i \geq PO_n\} \quad (9)$$

The fuzzy intervals or fuzzy numbers are defined on these sets.

SA AFC makes the quantification on this sum:

$$x_i = x(t) + h(t) \quad (10)$$

where x_i is the i th value of input sum into the flash A/D converter, i is the serial number of quantification in the measurement cycle, $x(t)$ is the value of the variable which is fuzzyficated in the moment of sampling, $h(t)$ is the value of random variable of uniform distribution in the moment of sampling.

Membership function of fuzzy sets (A_j) is defined depending on the relative frequency of appearing of the measuring result in (A_j) during the measuring cycle.

$$\mu_{A_j}(x) = \frac{a_j}{N} \quad (11)$$

where: j is the serial number of the fuzzy set, ($j = 1, 2, \dots, (n+1)$), a_j is the number of appearing value on the output of the fuzzy set A_j , N is the total quantification number during the measuring cycle.

The following limitation is in force for h :

$$h \leq \min(PO_j - PO_{j-1}) \quad \text{where } j = 2, \dots, n. \quad (12)$$

where h must be smaller or equal to the minimum difference between the two neighboring deciding thresholds.

The elements of continuous set are ordered to the membership function values as a secondary fuzzy set, constructed a type-2 fuzzy environment.

If we deal with system with great number of inputs and with limited capability, not only for measurements, but for processing as well, we can use this system more efficiently if we apply above mentioned idea: if processing system works longer on specific input the information is more detailed and more reliable, in the opposite case information is rough and less reliable. Similarly to human reasoning, system can concentrate on most significant input and most significant information is processed. For other inputs, system only confirms that they are under control.

The appearance of the signal in the first instant of quantization indicates rough estimation of specific input and it can define processing time interval. If estimation tells that input is under control, we can process immediately the next input. But if estimation tells that input is not under control, system pay more attention (longer processing time) to go back under control.

Conclusions

The T2 FS are of primary importance in all preliminary works in order to later construct inference mechanism based on this family of fuzzy operators and T2 fuzzy logic.

A possible application of type-2 fuzzy sets, related to signal processing and measure method SAADK was represented. The main advantages of the stochastic adding A/D conversion compared to standard digital instrumentation are its extremely simple hardware and simple implementation of parallel measurements, towards the possibility to trade speeds for accuracy. This instrument can be fast and less accurate, or slow and very accurate, depending on the choice of frequency of reading its output, therefore the membership on the universe X is time-dependant and with random uncertainties, it means suitable for representing it as T2 FS. The hardware realization of the problem is based only on the min and max operators, therefore distance based operators, which are also defined using min and max, making them applicable in the process of realization. It is an opportunity to use simply hardware realization of the operators and approximate reasoning process in the control problems based on the SAADK method.

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References

- [1] J. M. Mendel: Type-2 Fuzzy Sets: Some Questions and Answers, IEEE Neural Network Society, Aug. 2003

- [2] J. M. Mendel, R. I. B. John: Type-2 Fuzzy Sets Made Simple, IEEE Transactions on Fuzzy Systems, 10/2, pp. 117-127, 2002
- [3] K. Nagy, V. Vujcic, Z. Mitrovic, M. Takács: Fuzzyfication and Measurement Using Stochastic Approach, SISY 2007 Conference
- [4] Rudas, I. J.: Evolutionary Operators: New Parametric Type Operator Families, Fuzzy Sets and Systems 23, pp. 149-166, 1999
- [5] Tuhran Ozen, J. M. Garibaldi: Investigating Adaptation in Type-2 Fuzzy Logic System, Applied to Umbilical Acid-Base assessment
- [6] Zadeh, L. A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning – 1, Information Sciences, Vol. 8. pp. 199-249, 1975
- [7] Mendel, J. M.: Uncertainty, Fuzzy Logic, and Signal Processing, Signal Processing, Vol. 80, Issue 6, 2000